



Progression in calculations

Year 1

National curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- Add and subtract one-digit and two-digit numbers to 100, including zero (N.B. Year 1 N.C. objective is to do this with numbers to 20).
- Add and subtract numbers using concrete objects, pictorial Representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, 2 two-digit numbers; add 3 one-digit numbers (Year 2).
- Represent and use number bonds and related subtraction facts within 20.
- Given a number, identify 1 more and 1 less.
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot (Year 2).
- Recognise the inverse relationship between addition and subtraction and use this to solve missing number problems (Year 2).








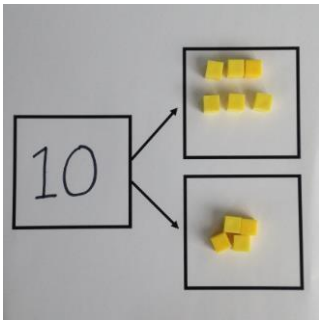
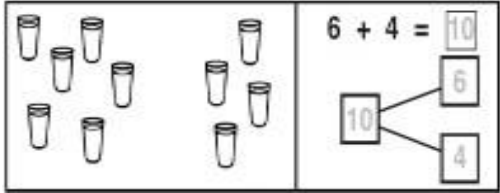
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equal (=) signs.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial Representations, and missing number problems, such as $7 = \square - 9$.
- Solve problems with addition and subtraction:
 - Using concrete objects and pictorial Representations, including those involving numbers, quantities and measures
 - Applying their increasing knowledge of mental methods

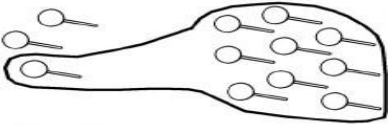


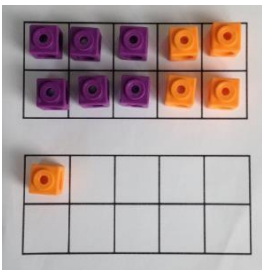

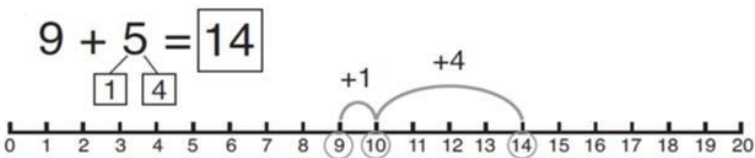
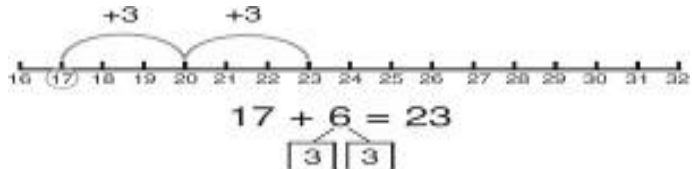
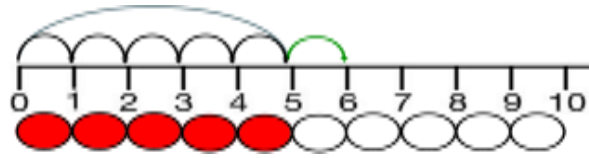

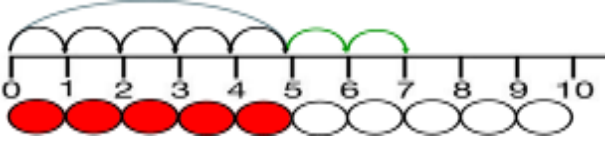
Teachers should refer to the definitions and guidance on the [structures for addition and subtraction](#) to provide a range of appropriate real-life contexts for calculations.



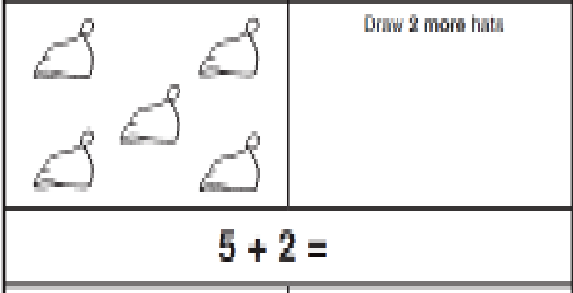

Year 1 Addition

Strategy & guidance	Representations
<p>Count all</p> <p><i>Joining two groups and then recounting all objects using one-to-one correspondence</i></p>	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $3 + 4 = 7$   </div> <div style="text-align: center;">  $5 + 3 = 8$ </div> </div>
<p>Counting on</p> <p><i>As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater number is more efficient.</i></p>	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $8 + 1 = 9$   </div> <div style="text-align: center;"> $15 = 12 + 3$   </div> </div>
<p>Part-whole</p> <p><i>Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse relationship between them.</i></p> <p><i>This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.</i></p>	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  $10 = 6 + 4$ $10 - 6 = 4$ $10 = 4 + 6$ $10 - 4 = 6$ </div> </div>



Strategy & guidance	Representations
<p>Regrouping ten ones to make ten</p> <p><i>This is an essential skill that will support column addition later on.</i></p>	 <p>$3 + 9 =$</p>  <p>$3 + 9 = 12$</p> 
<p>'Make ten' strategy</p> <p><i>Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.</i></p> <p><i>The colours of the beads on the bead string make it clear how many more need to be added to make ten.</i></p> <p><i>Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.</i></p>	<p>$6 + 5 = 11$</p>  <p>$4 + 9 = 13$</p>  <p>$9 + 5 = 14$</p>  <p>$17 + 6 = 23$</p> 
<p>Adding 1, 2, 3 more</p> <p><i>Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as; '1 more than 5 is equal to 6.'</i></p>	<p>1 more than 5 $5 + 1 = 6$</p>  <p>2 more than 5 $5 + 2 = 7$</p>  


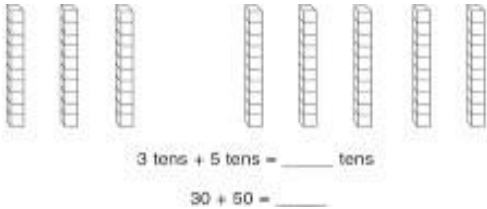
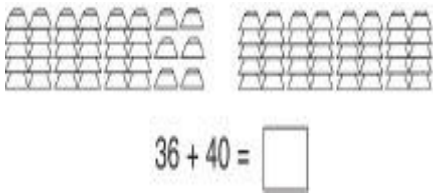


Strategy & guidance	Representations
<p>'2 more than 5 is equal to 7.'</p> <p>'8 is 3 more than 5.'</p> <p>Over time, pupils should be encouraged to rely more on their number bonds knowledge than on counting strategies.</p>	
<p>Adding three single digit numbers (make ten first)</p> <p>Pupils may need to try different combinations before they find the two numbers that make 10.</p> <p>The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.</p> <p>The second bead string shows 4, 6 and then 7.</p> <p>The final bead string shows how they have now been put together to find the total.</p>	 $ \begin{aligned} (4) + 7 + (6) &= \boxed{10} + \boxed{7} \\ &= \boxed{17} \end{aligned} $



Strategy & guidance	Representations
<p>Partitioning to add (no regrouping)</p> <p><i>Place value grids and Dienes blocks could be used as shown in the diagram before moving onto pictorial Representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.</i></p> <p><i>When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.</i></p>	<p style="text-align: center;">$24 + 13 = 37$</p> <p style="text-align: center;">$24 + 13 = 37$</p>
<p>Introducing column method for addition, regrouping only</p> <p><i>Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.</i></p> <p><i>See additional guidance on MyMastery for extra guidance on this strategy.</i></p>	<p style="text-align: center;">$24 + 17$</p> <div style="text-align: center;"> <p>Tens Ones</p> $\begin{array}{r} 24 \\ + 17 \\ \hline 41 \end{array}$ <p>First add the ones. group 10 ones to 1 ten. Next add the tens.</p> </div>



Strategy & guidance	Representations
<p>Adding multiples of ten</p> <p><i>Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a ten and not a one that is being added and they need to understand that a '2' digit in the tens column has a value of twenty.</i></p> <p><i>It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.</i></p>	<p style="text-align: center;">50 = 30 + 20</p>   

Year 1 Subtraction

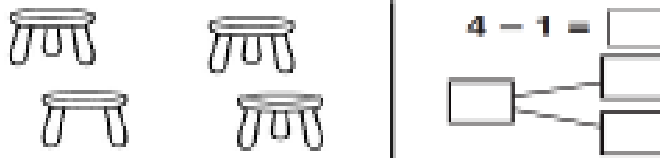
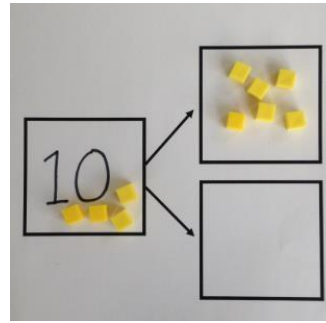
Strategy & guidance	Representations
<p>Taking away from the ones</p> <p><i>When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.</i></p>	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;">$7 - 3 = 4$</p> </div> <div style="text-align: center;"> <p style="text-align: center;">$15 - 3 = 12$</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p style="text-align: center;">$6 - 2 = 4$</p> </div> <div style="text-align: center;"> <p style="text-align: center;">$28 - 4 =$</p> </div> </div>
<p>Counting back</p> <p><i>Subtracting 1, 2, or 3 by counting back</i></p> <p><i>Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.</i></p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p style="text-align: center;">$4 = 6 - 2$</p> </div> <div style="text-align: center;"> <p style="margin-bottom: 5px;">$16 - 2 = 14$</p> </div> </div>



Part-part-whole

Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole. They then remove what is being taken away from the whole and place it on one of the parts. The remaining cubes are the other part and also the answer. These can be moved into the second part space.

$$10 - 6 = 4$$

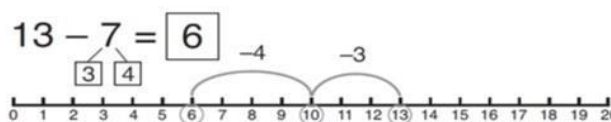
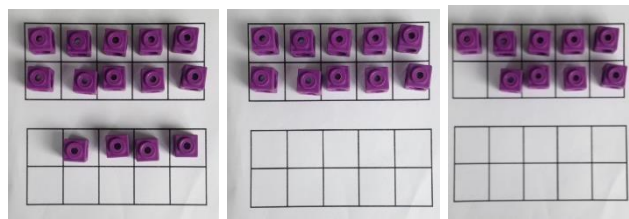


Make ten strategy

To subtract a 1-digit number from a 2-digit number.

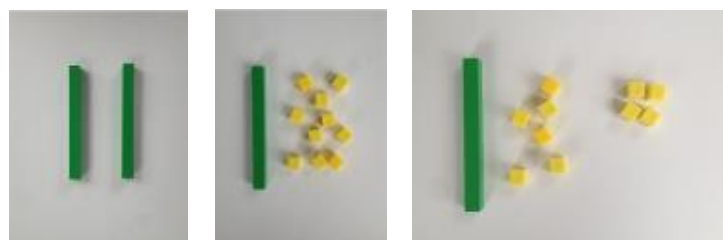
Pupils identify how many need to be taken away to make ten first, partitioning the number being subtracted. Then they take away the rest to reach the answer.

$$14 - 5 = 9$$



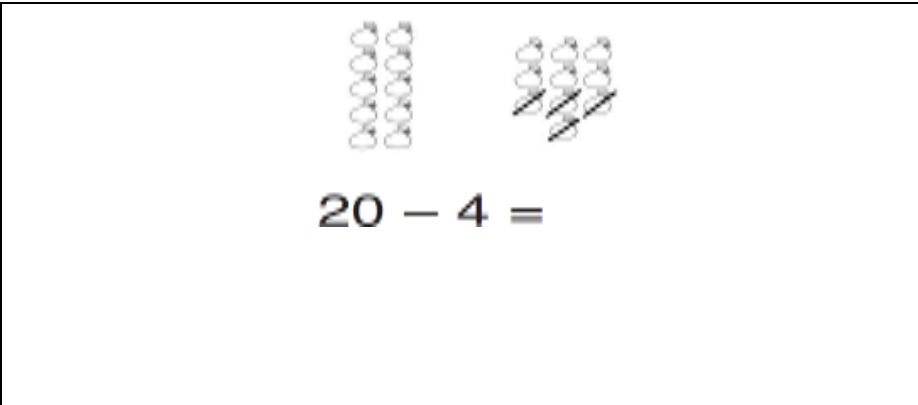
Regroup a ten into 10 ones

After the initial introduction, the Dienes blocks should be placed on a place value chart to support place



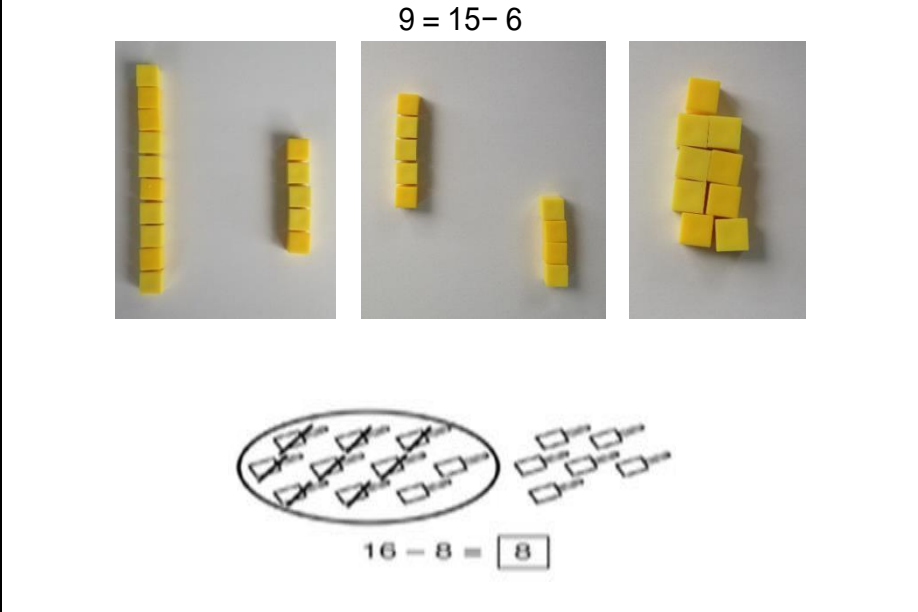


value understanding. This will support pupils when they later use the column method.



Taking away from the tens

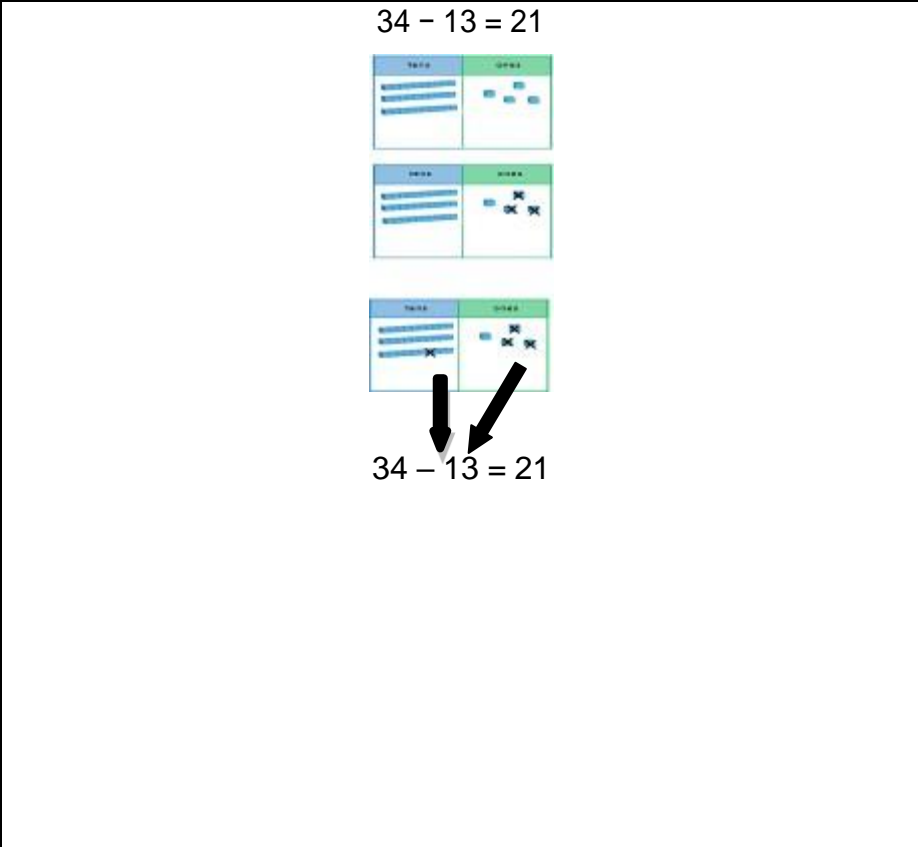
Pupils should identify that they can also take away from the tens and get the same answer. This reinforces their knowledge of number bonds to 10 and develops their application of number bonds for mental strategies.



Partitioning to subtract without regrouping

Dienes blocks on a place value chart (developing into using images on the chart) could be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1.

When not regrouping, partitioning is a mental strategy and does not need formal recording in

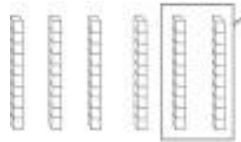


columns. This representation prepares them for using column subtraction with formal recording.

Subtracting multiples of ten

Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a **ten** not a one that is being taken away.

$$40 = 60 - 20$$



$$6 \text{ tens} - 2 \text{ tens} = \text{ } \text{tens}$$

$$60 - 20 = \text{ }$$

$$38 - 10 = 28$$

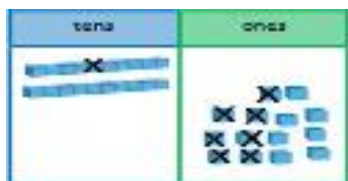


$$38 - 10 = \square$$

Column method with regrouping

This example shows how pupils should work practically when being introduced to this method. There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2. See additional guidance on MyMastery to support with this method.

$$34 - 17 = 17$$







National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial Representations and arrays with the support of the teacher.

Teachers should refer to definitions and guidance on the [structures for multiplication and division](#) to provide a range of appropriate real-life contexts for calculations.

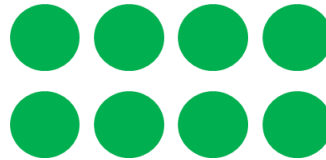
Year 1 Multiplication

Strategy & guidance	Representations
<p>Skip counting in multiples of 2, 5, 10 from zero</p> <p><i>The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.</i></p> <p><i>Count the groups as pupils are skip counting.</i></p> <p><i>Number lines can be used in the same way as the bead string.</i></p> <p><i>Pupils can use their fingers as they are skip counting.</i></p>	<div style="text-align: center;">  <p>5, 10, 15, 20</p>  <p>2, 4, 6, 8</p> </div>

Making equal groups and counting the total

In Y1 emphasis should be placed on the vocabulary used alongside the representation. So this picture could represent 2 groups of 4 or 4 twice. Pupils will build familiarity with the array representation and language of equal groups. .

Pupils will not use formal multiplication and division equations until Y2.



*There are four **equal groups** of two. There are eight altogether. The **whole** is eight.*

Solve multiplication problems using concrete or pictorial Representations and skip counting.

Pupils explore finding the total number of objects arranged in equal groups.

They begin by doing this with concrete items then move on to pictorial Representations of the items before relating this to familiar

Representations such as the array and part whole model.

Language of equal groups should be used throughout so that pupils build an understanding of multiplicative structures.

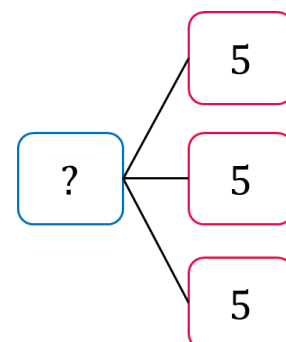
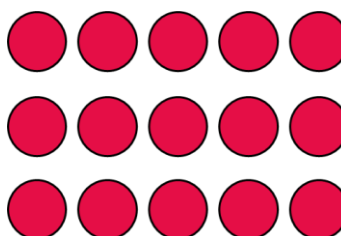
How many are there altogether?



There are four equal groups. There are five pens in each group.

5, 10, 15, 10



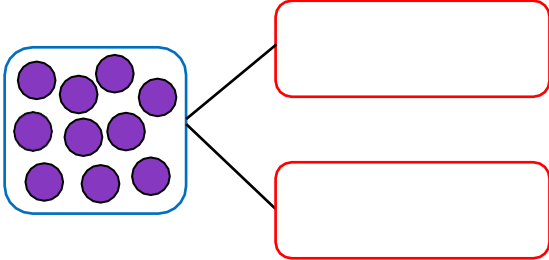
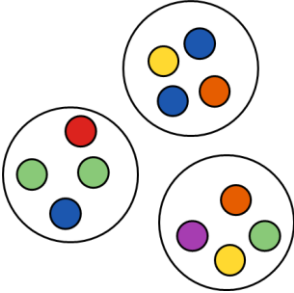
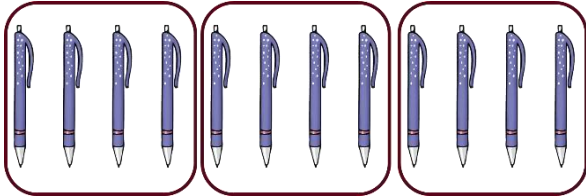
The whole is 20. There are 20 pens altogether.



There are three equal groups of five.

5, 10, 15. The whole is 15.

Year 1 Division

Strategy & guidance	Representations
<p>Sharing objects into groups</p> <p>(Partitive division)</p> <p><i>Pupils should become familiar with division problems. Language of sharing into equal groups should be used.</i></p> <p><i>The division symbol and formal equations are not introduced until Year 2.</i></p>	<p style="text-align: center;">Share ten into two equal groups.</p> <div style="display: flex; justify-content: space-around;">   </div> <div style="text-align: center;">  <p style="text-align: center;"><i>The whole is ten.</i></p> <p style="text-align: center;"><i>There are two equal groups.</i></p> <p style="text-align: center;"><i>Each groups has five.</i></p> </div>
<p>Grouping objects</p> <p>(Quotative division)</p> <p><i>Pupils become familiar with grouping into equal groups. They do this firstly concretely, then pictorially by drawing rings around pictorial representations.</i></p>	<div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>One, two, three, four. One, two, three, four. One, two, three, four.</p> <p>All the counters have been divided equally into groups of four.</p> <p style="text-align: center;">There are three groups. 12 shared equally into groups of four makes three groups.</p> </div> </div> <div style="text-align: center; margin-top: 20px;">  </div>



Progression in calculations Year 2

National Curriculum objectives linked to addition and subtraction

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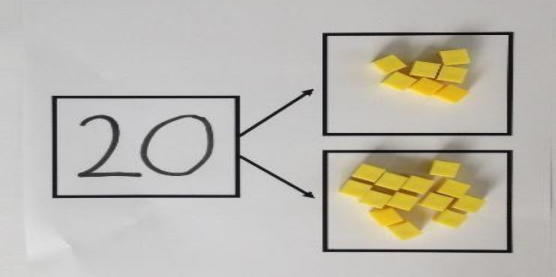
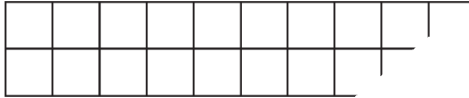
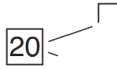
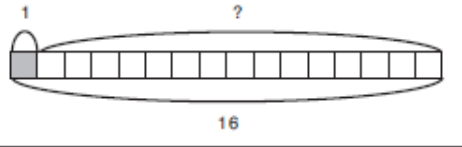
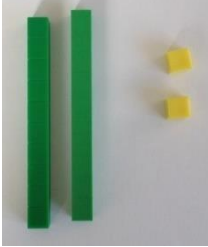
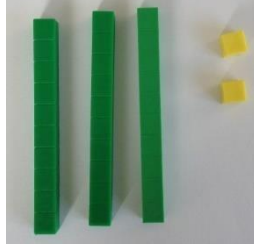
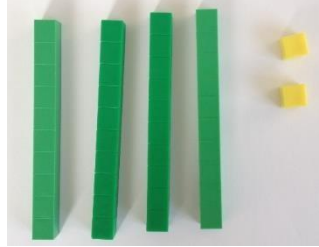
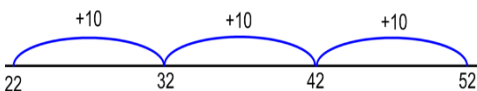
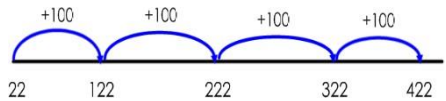
- Add and subtract numbers using concrete objects, pictorial Representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; 2 two-digit numbers; adding three one-digit numbers.
- Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds (Year 3).
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Find 10 or 100 more or less than a given number (Year 3).
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction (Year 3).

The following objectives should be planned for lessons where new strategies are being introduced and developed:




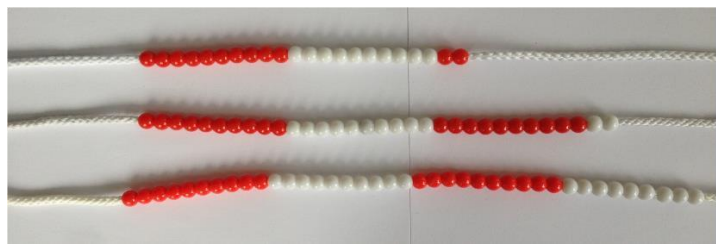


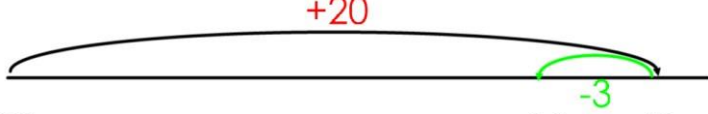
- Solve problems with addition and subtraction: using concrete objects and pictorial Representations, including those involving numbers, quantities and measures; apply increasing knowledge of mental and written methods.
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction. (Year 3)

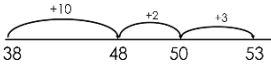
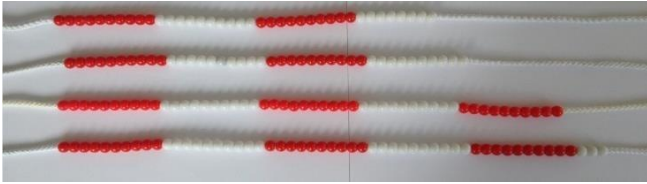
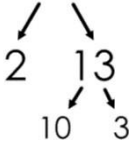
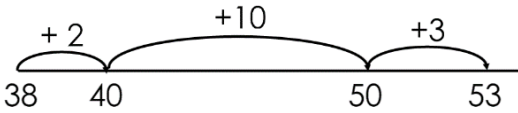
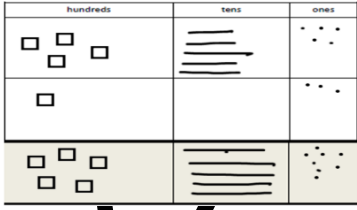
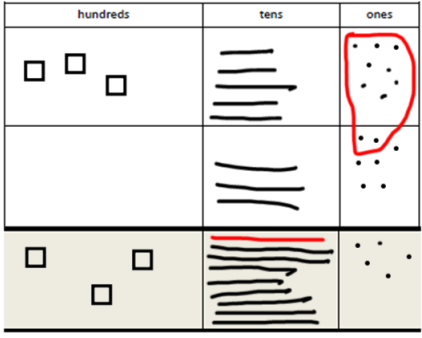
Teachers should refer to the definitions and guidance on the [structures for addition and subtraction](#) to provide a range of appropriate real-life contexts for calculations.

Year 2 Addition

Strategy & guidance	Representations
<p>Part-part-whole</p> <p><i>Pupils explore the different ways of making 20. They can do this with all numbers using the same Representations.</i></p> <p><i>This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.</i></p>	<div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="text-align: center;">  </div> <div style="text-align: right;"> $20 = 17 + 3$ $20 = 3 + 17$ $20 - 3 = 17$ $20 - 17 = 3$ </div> </div> <div style="text-align: center; margin-top: 20px;">  </div> <div style="text-align: center; margin-top: 20px;">  </div> <div style="text-align: center; margin-top: 20px;"> $\square + \square = \square$ \square </div> <div style="text-align: center; margin-top: 20px;"> <hr style="width: 100%;"/> $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$ </div> <div style="text-align: center; margin-top: 20px;">  </div>
<p>Counting on in tens and hundreds</p>	<div style="display: flex; justify-content: space-around; align-items: center;">    </div> <div style="text-align: center; margin-top: 20px;">  </div> <div style="text-align: center; margin-top: 20px;">  </div>



Strategy & guidance	Representations
<p>Using known facts to create derived facts</p> <p><i>Dienes blocks should be used alongside pictorial and abstract Representations when introducing this strategy.</i></p>	<div style="display: flex; justify-content: space-between;"> <div style="text-align: center;">    </div> <div style="text-align: left;"> <p>$3 + 4 = 7$</p> <p><i>leads to</i></p> <p>$30 + 40 = 70$</p> <p><i>leads to</i></p> <p>$300 + 400 = 700$</p> </div> </div>
<p>Partitioning one number, then adding tens and ones</p> <p><i>Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.</i></p>	 <div style="text-align: center;"> <p>$+10$ $+7$</p>  <p>22 32 39</p> <p>$22 + 17 = 39$</p> </div>
<p>Round and adjust (sometimes known as a compensating strategy)</p> <p><i>Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.</i></p>	 <div style="text-align: center;"> <p>$+20$</p>  <p>22 39 42</p> <p>$22 + 17 = 39$</p> </div>

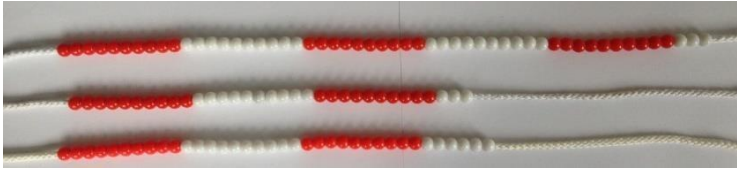
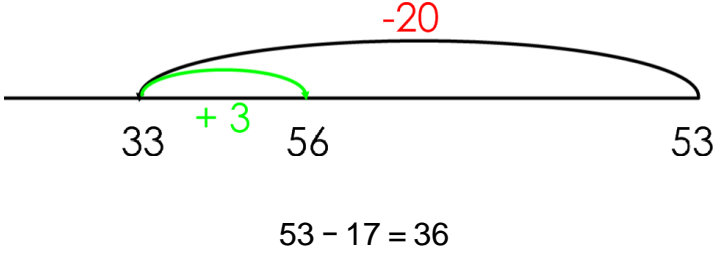

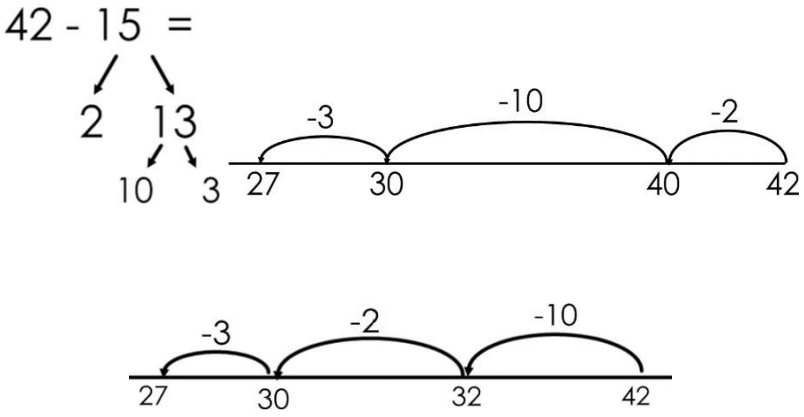
Strategy & guidance	Representations																
<p>Make ten strategy</p>  <p>How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.</p> <p>It relies on an understanding that numbers can be partitioned in different ways in order to easily make a multiple of ten.</p>	 $38 + 15 =$  																
<p>Partitioning to add without regrouping</p> <p>As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3-digit numbers but do not record a formal written method if there is no regrouping.</p>	 $455 + 103 = 558$																
<p>Column method with regrouping</p> <p>Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.</p> <p>As in Year 1, the focus is to develop a strong understanding of place value.</p>	<table border="1" data-bbox="598 1556 917 1780"> <thead> <tr> <th></th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td></td> <td>3</td> <td>5</td> <td>8</td> </tr> <tr> <td>+</td> <td></td> <td>3</td> <td>7</td> </tr> <tr> <td></td> <td>3</td> <td>9</td> <td>5</td> </tr> </tbody> </table> 		hundreds	tens	ones		3	5	8	+		3	7		3	9	5
	hundreds	tens	ones														
	3	5	8														
+		3	7														
	3	9	5														



Year 2 Subtraction

Strategy & guidance	Representations
<p>Counting back in multiples of ten and one hundred</p>	<p style="text-align: center;"> -10 -10 $\overbrace{\hspace{10em}}$ 75 85 95 </p> <p style="text-align: center;"> -100 -100 $\overbrace{\hspace{15em}}$ 750 850 950 </p>
<p>Using known number facts to create derived facts</p> <p><i>Dienes blocks should be used alongside pictorial and abstract</i></p> <p><i>Representations when introducing this strategy, encouraging pupils to apply their knowledge of number bonds to add multiples of ten and 100.</i></p>	<div style="display: flex; justify-content: space-between; align-items: center; width: 100%;"> <div style="text-align: right;"> $8 - 4 = 4$ <i>leads to</i> $80 - 40 = 40$ <i>leads to</i> $800 - 400 = 400$ </div> </div>
<p>Subtracting tens and ones</p> <p><i>Pupils must be taught to partition the second number for this strategy as partitioning both numbers can lead to errors if regrouping is required.</i></p>	<p style="text-align: center;">$53 - 12 = 41$</p> <p style="text-align: center;"> -2 -10 $\overbrace{\hspace{10em}}$ 41 43 53 </p>



Strategy & guidance	Representations
<p>Round and adjust (sometimes known as a compensating strategy)</p> <p><i>Pupils must be taught to round the number that is being subtracted.</i></p> <p><i>Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.</i></p>	 
<p>Make ten</p> <p><i>How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.</i></p> <p><i>It relies on an understanding that numbers can be partitioned in different ways in order to subtract to a multiple of ten.</i></p> <p><i>Pupils should develop an understanding that the parts can be added in any order.</i></p>	 <p>$42 - 15 =$</p> 

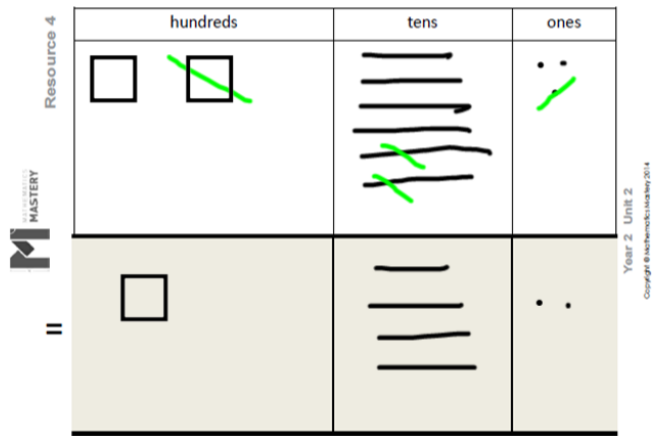


Strategy & guidance

Representations

Partitioning to subtract without regrouping

As in Year 1, the focus is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial. Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.



$$263 - 121 = 142$$

Column method with regrouping

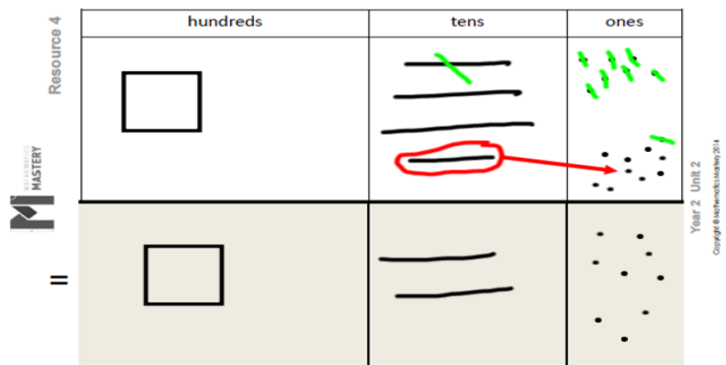
The focus for the column method is to develop a strong understanding of place value and concrete manipulatives should be used alongside.

Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens). E.g. $232 - 157$ and are given plenty of practice using concrete and pictorial representations alongside their formal written methods, ensuring that important steps are not missed in the recording.

Caution should be exercised when

hundreds tens ones

$$\begin{array}{r} 1\overset{3}{4}17 \\ - 18 \\ \hline 129 \end{array}$$





Strategy & guidance	Representations
<i>introducing calculations requiring 'regrouping to regroup' (e.g. 204 – 137) ensuring ample teacher modelling using concrete manipulatives and images.</i>	



National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

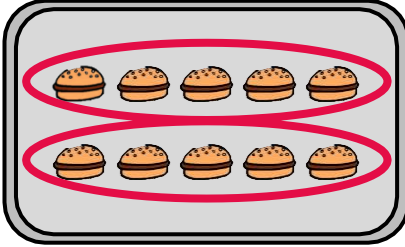
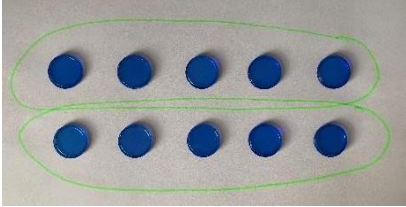
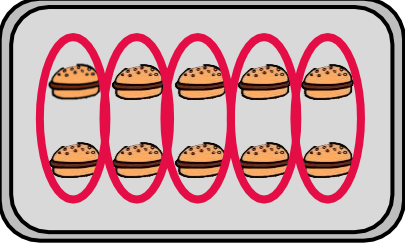
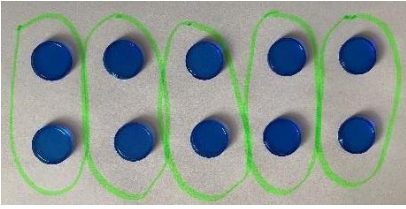
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Recall and use multiplication and division facts for the 3 and 4 multiplication tables (Year 3).
- Show that multiplication of two numbers can be done in any order (commutative) but division of one number by another cannot.


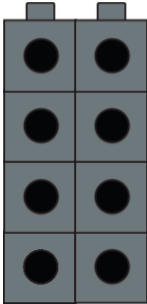

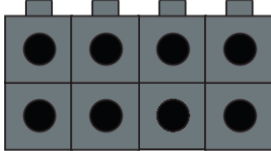
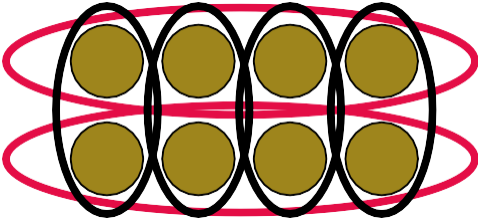
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equal ($=$) signs.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

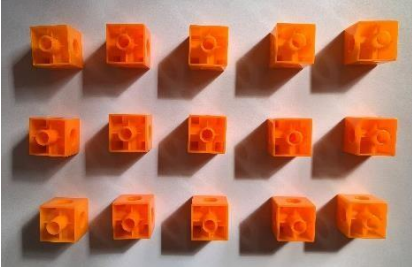
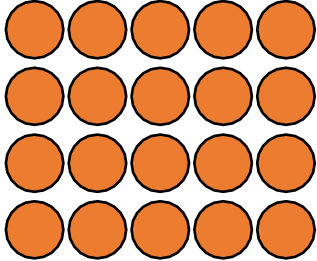
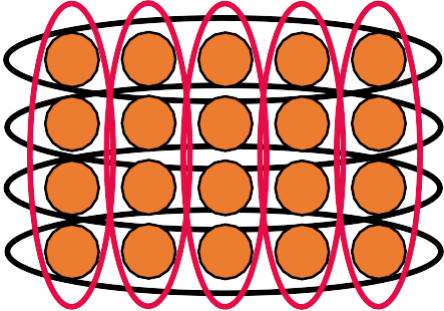
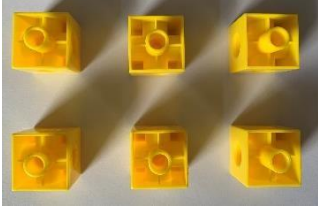
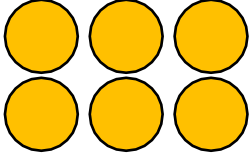
Teachers should refer to definitions and guidance on the [structures for multiplication and division](#) to provide a range of appropriate real-life contexts for calculations.

Year 2 Multiplication

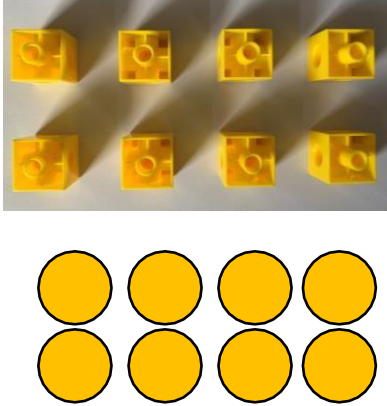
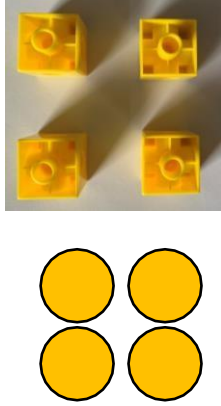
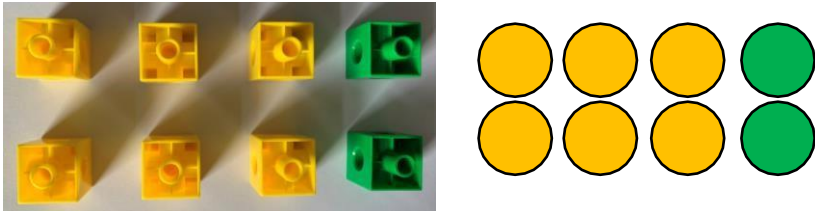
Strategy & guidance	Representations
<p>Making and describing equal and unequal groups</p> <p><i>Concrete manipulatives and images of objects begin to be organised into rows or columns of equal length thus creating a rectangular array. Pupils should be encouraged to describe what they can see referring to equal grouping and encourage flexibility in the two ways the array can be described.</i></p> <p><i>It is important to discuss with pupils how arrays can be useful.</i></p> <p><i>Pupils move towards attaching the abstract notation of multiplication and division, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.</i></p> <p><i>The relationship between multiplication and division also begins to be demonstrated.</i></p>	<div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p style="text-align: center;">“I can see two equal groups, each with a value of five. The whole is ten.”</p> <p style="text-align: center;">$2 \times 5 = 10$</p> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;">   </div> <p style="text-align: center;">“I can see five equal groups, each with a value of two. The whole is ten.”</p> <p style="text-align: center;">$5 \times 2 = 10$</p> <p style="text-align: center;">Pupils should be encouraged to think flexibly when writing the abstract equation seeing the one array as a representation for both equations.</p>

Strategy & guidance	Representations
<p>Drawing around equal groups to show multiplication is commutative</p> <p><i>Pupils build on their understanding that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</i></p> <p><i>Encourage pupils to compare two arrays representing the same problem and identify that the whole remains the same by rotating the array to sit one on top of the other.</i></p> <p><i>Describing and annotating the one array to show the different ways of describing the equal groups supports their understanding.</i></p>	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <p style="border: 1px solid red; border-radius: 15px; padding: 5px; width: 150px; margin: 10px auto;">Robin has two bags with four sweets in each.</p> </div> <div style="text-align: center;">  </div> <div style="text-align: center;">   <p style="border: 1px solid red; border-radius: 15px; padding: 5px; width: 150px; margin: 10px auto;">Ishmael has four bags with two sweets in each.</p> </div> </div> <p style="text-align: center;"><i>“I can see two equal parts of four. The whole is eight.”</i></p> <p style="text-align: center;">Or</p> <p style="text-align: center;"><i>“I can see four equal parts of two. The whole is eight.”</i></p> <p style="text-align: center;">$2 \times 4 = 8$ and $4 \times 2 = 8$</p> <div style="text-align: center;">  </div>



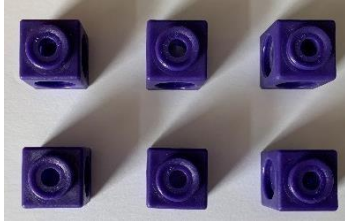
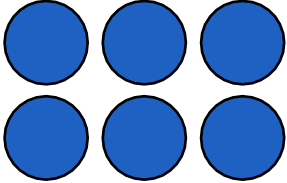
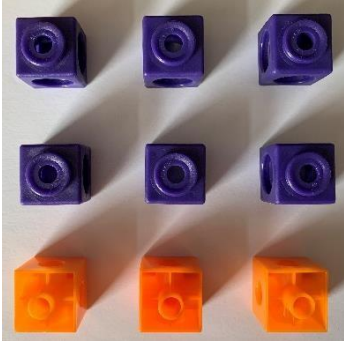
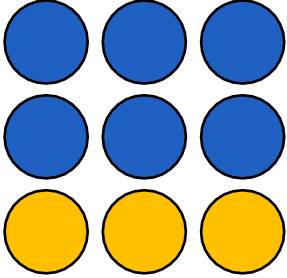
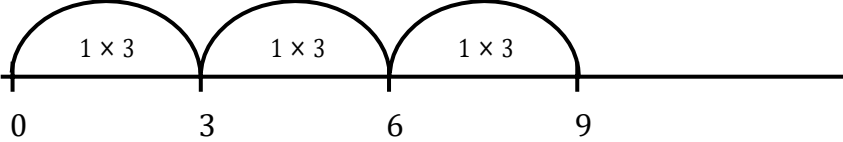
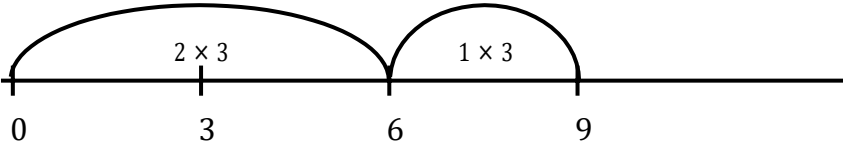
Strategy & guidance	Representations
<p>Use of an array to establish the inverse relationship between multiplication and division</p> <p><i>Pupils use arrays of manipulatives and images to represent multiplicative contexts where all information is provided. Pupils should be encouraged to use part-whole language to describe and create an array focusing on the structure.</i></p> <p><i>This link should be made explicit from early on so that pupils develop an early understanding of the relationship between multiplication and division.</i></p> <p><i>Pupils record the four facts that can be derived from the one array; two multiplication and two division.</i></p>	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid red; border-radius: 15px; padding: 10px; width: 45%;"> <p>There are five tables. Each table seats four children. 20 children can sit down.</p> </div> <div style="border: 1px solid red; border-radius: 15px; padding: 10px; width: 45%;"> <p>20 children need to sit down. Each table seats four children. There are five tables.</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;">   </div> <p style="text-align: center;">“There are five equal parts, each with a value of four. The whole is 20.”</p> <p style="text-align: center;">“I know the whole is 20 and the value of each part is four. The number of parts needed is five.”</p> <div style="text-align: center; margin-top: 20px;">  <p>$4 \times 5 = 20$ and $5 \times 4 = 20$</p> <p>$20 \div 4 = 5$ and $20 \div 5 = 4$</p> </div>
<p>Adding and subtracting equal groups to support skip counting</p> <p><i>Pupils apply their knowledge of equal groups and apply this to skip counting to help find the totals of</i></p>	<div style="display: flex; justify-content: space-around; margin-bottom: 20px;">   </div> <p style="text-align: center;">“There are three equal groups of two. The whole is six.”</p> <p style="text-align: center;">$2 \times 3 = 6$</p>



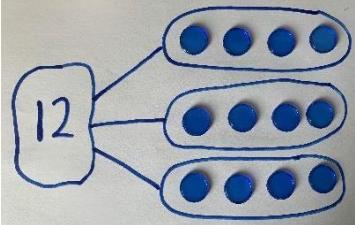

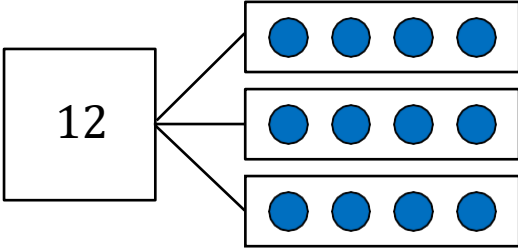
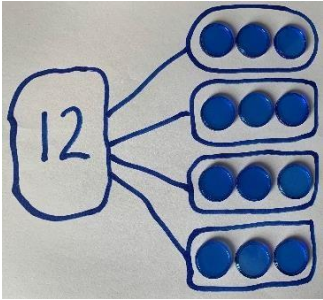

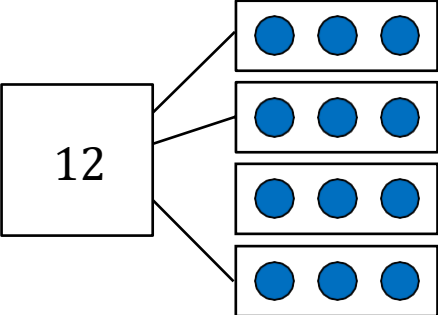
Strategy & guidance	Representations
<p><i>repeated additions with 2x, 5x and 10x</i></p> <p><i>The purpose is to recognise the relationship between the number of groups and the group size therefore ensure pupils are clear on the consistent factor being the explored.</i></p> <p><i>Pupils should always describe the array before then attaching the abstract equation to it.</i></p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p><i>“I’m going to add another equal group of two. There are four equal groups of two. The whole is eight.”</i></p> <p>$2 \times 4 = 8$</p> </div> <div style="text-align: center;">  <p><i>“I’m going to remove an equal group of two. There are two equal groups of two. The whole is four.”</i></p> <p>$2 \times 2 = 4$</p> </div> </div>
	<div style="text-align: center;">  <p><i>“I can see three groups of two plus one more group of two.”</i></p> <p>$2 \times 3 + 2 \times 1$</p> </div>

Strategy & guidance	Representations
<p>Halving and doubling to derive new multiplication facts</p> <p><i>Pupils apply their knowledge of halving and relate this to doubling as inverse operations, connecting halving to dividing by two and doubling as multiplying by two.</i></p> <p><i>At this stage they double the 2x table facts to derive the 4x table facts and should be encouraged to focus in on the similarities and differences between the arrays and the relationship common factor and the multiplier.</i></p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p style="text-align: center;">“The whole is eight. Eight shared between two equal groups is equal to four. One half of eight is equal to four.”</p> <p style="text-align: center;">$8 \div 2 = 4$</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <div style="text-align: center; margin-top: 10px;"> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid red; padding: 5px; display: inline-block;">This array represents 1×4.</div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;"> </div> </div> <div style="text-align: center; margin-top: 10px;"> <div style="border: 1px solid red; padding: 5px; display: inline-block;">This array now represents 2×4.</div> </div> <div style="text-align: center; margin-top: 10px;"> </div> <p style="text-align: center;">“There are two parts, each with a value of four. The whole is eight.”</p> <p style="text-align: center;">“$4 + 4$ is the same as 2×4 (two groups of four) which is the same as ‘double four’.”</p>

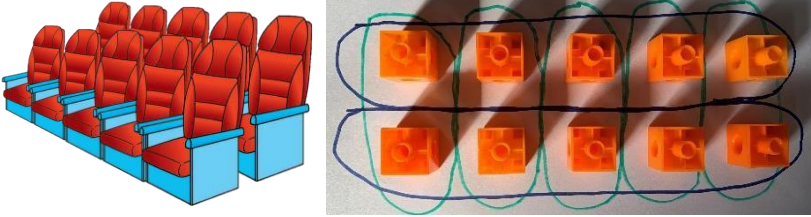


Strategy & guidance	Representations
<p>Representing known facts to derive new facts using and combining arrays and on a numberline (3×)</p> <p><i>Pupils build on their knowledge of adding equal groups, skip counting and repeated addition to support flexibility in understanding.</i></p> <p><i>Pupils create two arrays for two known facts, either using manipulatives or images, before combining to represent a derived fact from the three times table.</i></p> <p><i>Pupils move on to connect the arrays to jumps of equal value on a number line, connecting this to the abstract equations.</i></p>	<p>“I know this is 2×3 because there are two equal groups of three.”</p> <div style="display: flex; align-items: center; justify-content: space-around;">  <div style="text-align: center;"> $2 \times 3 = 6$ </div>  </div> <p>“To find out what 3×3 is we need to add another equal group of three.”</p> <div style="display: flex; align-items: center; justify-content: space-around;">  <div style="text-align: center;"> $2 \times 3 = 6$ $1 \times 3 = 3$ $3 \times 3 = 9$ </div>  </div> <p style="text-align: center;">$2 \times 3 + 1 \times 3 = 3 \times 3$</p> <p>“Three multiplied by three is equal to nine.”</p> <div style="text-align: center;">  <p>or</p>  </div>

Year 2 Division

Strategy & guidance	Representations
<p>Sharing objects into a given number of groups</p> <p>(Partitive division)</p> <p>Here, division is shown as sharing.</p> <p>Pupils use counters or cubes to create an array or a part-whole model, sharing the whole between the number of parts until there are no more objects left to sort.</p> <p>It is important to highlight that, as with multiplication, in division, the value of the parts should be equal.</p>	<p>There are 12 children altogether. There are three rows on the carpet. How many children will there be in each row?</p>   <p>“We know the whole is 12. We know there are three parts. We don’t know the value of the parts.”</p> 
<p>Grouping objects into sets of equal groups</p> <p>(Quotative division)</p> <p>Here, division is shown as grouping.</p> <p>Pupils use counters or cubes to create an array or a part-whole model, making equal groups to see how many can be made from the whole.</p> <p>It is important to highlight that, as with multiplication, in division, the value of the parts should be equal.</p>	<p>There are 12 children altogether. The children sit on the carpet in rows of three. How many rows will there be?</p>   <p>“We know the whole is 12. We know the value of each part is three. We don’t know the number of parts.”</p> 



Strategy & guidance	Representations
<p>Use of an array to establish the inverse relationship between multiplication and division and derive facts</p> <p><i>Pupils build on their understanding of division and an array to derive facts, connecting their fractional knowledge to division to derive six facts for each array.</i></p>	 <p>“I can see two equal groups of five which is equal to ten.”</p> <p>“I can see ten divided into five equal groups of ten.”</p> <p>“One half of ten is equal to five.”</p> <p>“One fifth of ten is equal to two.”</p> <p>$2 \times 5 = 10$ and $5 \times 2 = 10$</p> <p>$10 \div 2 = 5$ and $10 \div 5 = 2$</p> <p>$\frac{1}{2}$ of ten is equal to five</p> <p>$\frac{1}{5}$ of ten is equal to two</p>