Progression in calculations
Year 3

National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

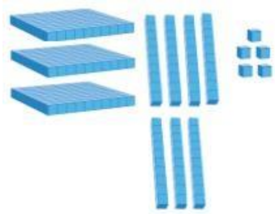
- add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add and subtract numbers with up to four digits, using formal written methods of columnar addition and subtraction (four digits is Year 4)
- find 10 or 100 more or less than a given number
- find 1 000 more or less than a given number (Year 4)
- estimate the answer to a calculation and use inverse operations to check answers

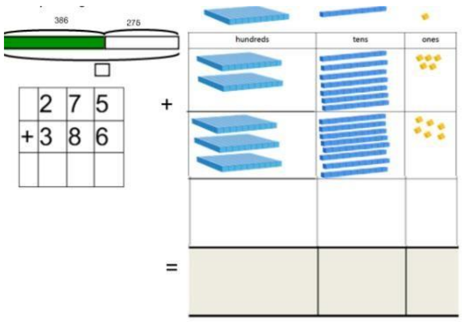
The following objectives should be planned for lessons where new strategies are being introduced and developed:

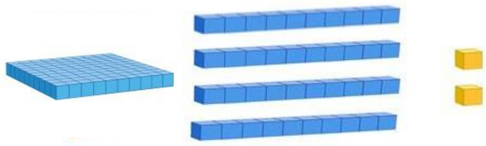
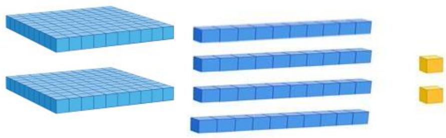
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Teachers should refer to definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

Year 3 Addition & Subtraction

Strategy & guidance	Representations
Add and subtract numbers mentally, including: • **a three-digit number and ones;** • **a three-digit number and tens;** • **a three-digit number and hundreds** *Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.* *To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):* • *known number facts within 20,* • *derived number facts,* • *'Make ten',* • *round and adjust* *See Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these strategies.* *It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.*	It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required, with numbers appropriate to the unit they are working on (3-digit numbers in Units 1 & 4; 4-digit numbers in Unit 13). However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images. No regrouping 345 + 30 274 - 50 1128 + 300 1312 - 300 326 + 342 856 - 724  I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens. 345 + 30 = 375. With some regrouping 416 + 25 232 - 5 383 + 130 455 - 216 611 + 194 130 - 40 1482 + 900 2382 - 500

Strategy & guidance	Representations
Written column method for calculations that require regrouping with up to 4-digits *Dienes blocks should be used alongside the pictorial Representations during direct teaching and can be used by pupils both for support and challenge. Place value counters can also be introduced at this stage.* *This work revises and reinforces ideas from Key Stage 1, including the focus on place value – see Year 2 exemplification.* *Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it. Asking them 'Can you think of a more efficient method?' will challenge them to apply their number sense / number facts to use efficient mental methods where possible.* *As in Year 2, pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping. In Year 3 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number facts, round and adjust etc.) may be likelier to produce an accurate solution.* *Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.* *See MyMastery for extra guidance on this strategy.*	As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.  5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one. <u>Regrouping (including multiple separate instances)</u> 672 + 136 734 – 82 468 + 67 831 - 76 275 + 386 435 – 188 '<u>Regrouping to regroup</u>' 204 – 137 1035 - 851

Strategy & guidance	Representations
Find 10, 100 more or less than a given number *As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (and, in the last unit of the Summer term, thousands), including use of concrete manipulatives and appropriate images.* *After initial teaching, this should be incorporated into transition activities and practised regularly.*	142 + 100 = 242  

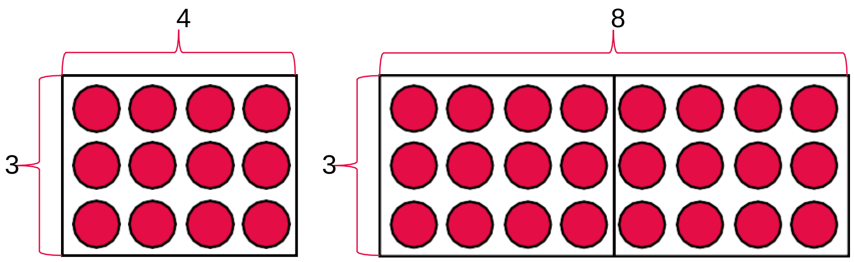
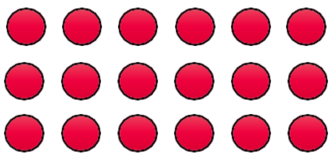
National Curriculum objectives linked to multiplication and division

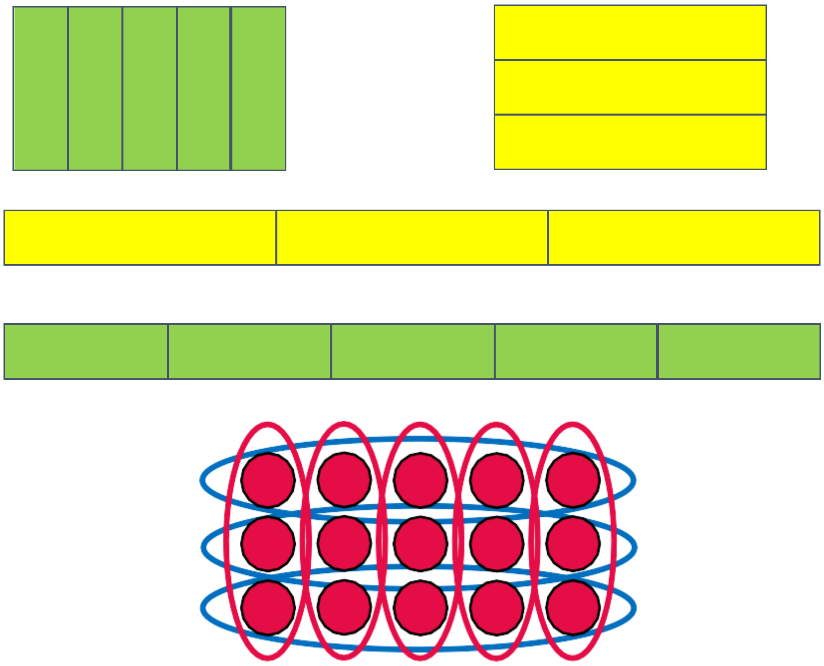
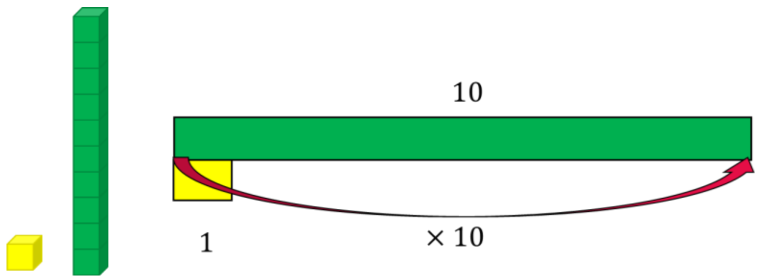
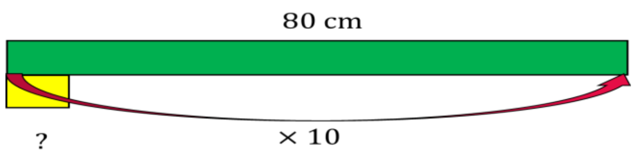
These objectives are explicitly covered through the strategies outlined in this document:

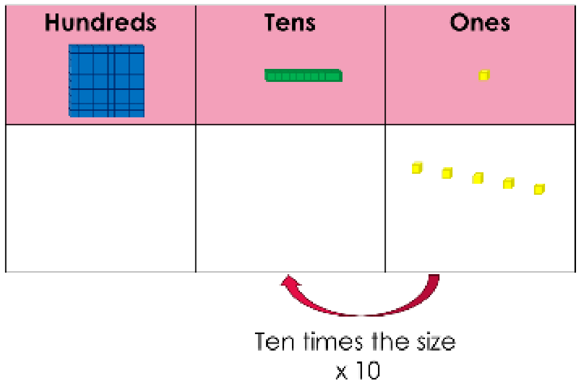
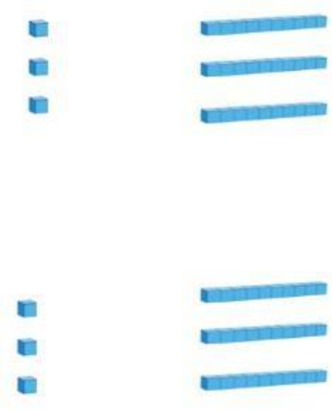
- count from 0 in multiples of 4, 8, 50 and 100
- recall and use multiplication and division facts for the 3, 4, and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects

Teachers should refer to definitions and guidance on the structures for multiplication and division to provide a range of appropriate real-life contexts for calculations.

Year 3 Multiplication

Strategy & guidance	Representations
Doubling to derive new multiplication facts *Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.* *Specifically, in Year 3, pupils will explore the link between the 4 and 8 times table* *This builds on the doubling strategy from Year 2.*	4 × 3 = 12 8 × 3 = 24  When we double one factor, the product will be double the size.
Skip counting in multiples of 2, 3, 4, 5, 8 and 10 *Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.*	 0 3 6 9 12 15 18 21 24 27 30  3, 6, 9, 12, 18

Strategy & guidance	Representations
Use of Cuisenaire with arrays and bar models to establish commutativity and inverse relationship between multiplication and division *In these contexts pupils are able to identify all the equations in a fact family.*	*Three groups of five is equal to five groups of three.*  3 × 5 = 5 × 3
Ten times the size *Pupils' work on this must be firmly based on concrete Representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.*	 *For every one I need to use a ten. 10 is ten times the size of one.* 

Strategy & guidance	Representations
Multiplying by 10 *When you multiply whole numbers by 10 this is equivalent to making a number 10 times the size.* *When you multiply by ten, each part is ten times the size. The ones become tens, the tens become hundreds, etc.* *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.*	 *5 made 10 times the size is 50.* *50 is ten times the size of 5.* *5 multiplied by ten is 50.*
Using known facts for multiplying by multiples of 10 *Pupils' growing understanding of place value allows them to make use of known facts to derive multiplications using scaling by 10.* *It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)*	$3 \times 2 = 6$ $30 \times 2 = 60$ 

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Multiplication of 2-digit numbers with partitioning (no regrouping)

Pupils should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.

Pupils may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.

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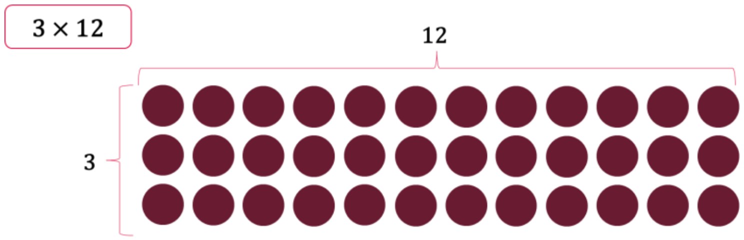

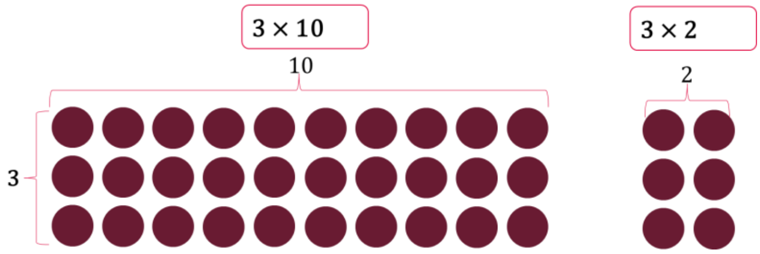

×	10	2
3		

×	10	2
3	30	6

3 × 12 = 36

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<tr><th>Strategy & guidance</th><th>Representations</th></tr>
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Multiplication of 2-digit numbers with partitioning (regrouping)

Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4.

Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.

Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.

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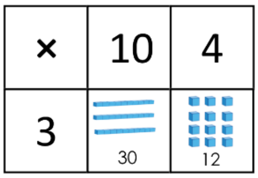

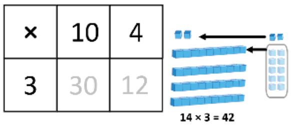

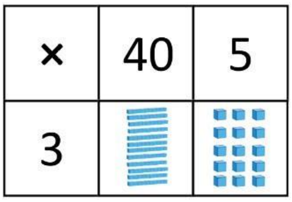

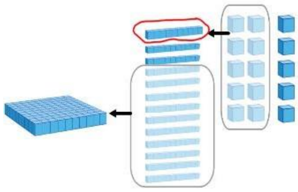

1) First, I need to partition my 2-digit number into tens and ones.

2) I need to multiply my ones by____. There are_____ones.

I can regroup my ones into____or I do not need to regroup my ones.

3) I need to multiply my tens by____. There are_____tens.

I can regroup my tens into____or I do not need to regroup.

4) I can add the tens and ones to get the product. ____ multiplied by____is____.

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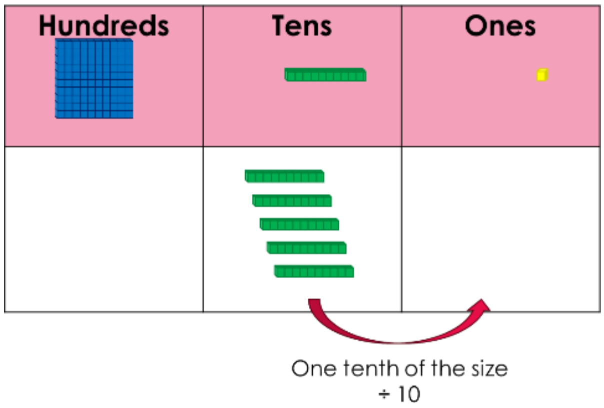
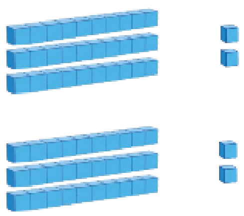
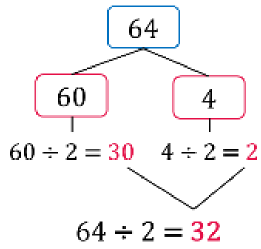
Year 3 Division

Strategy & Guidance	Representations
Dividing by 10 *When you divide by ten, each part is ten times smaller or one tenth of the sise. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.* *When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21*	
Dividing a 2-digit number by a 1-digit number (no regrouping) *Pupils use partitioning to divide a 2-digit number with no regrouping. This will be built upon in year 4 when pupils move towards short division.*	64 ÷ 2 =  

National curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

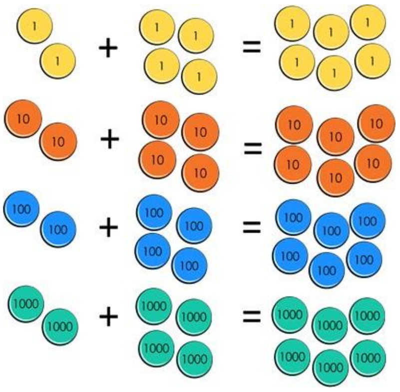
- add and subtract numbers with up to four digits, using the formal written methods of columnar addition and subtraction where appropriate
- find 1 000 more or less than a given number
- estimate and use inverse operations to check answers to a calculation

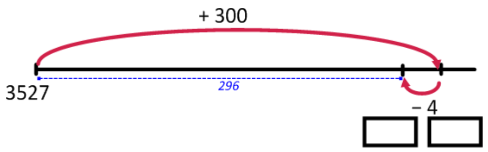
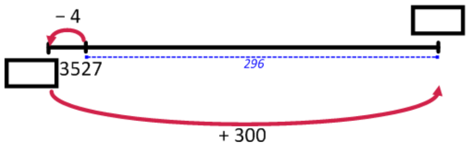
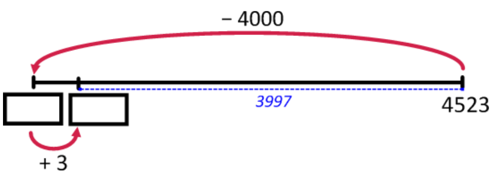
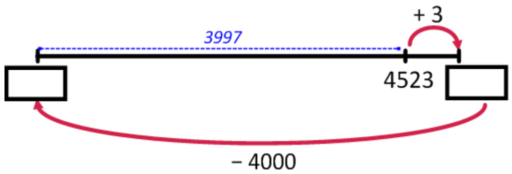
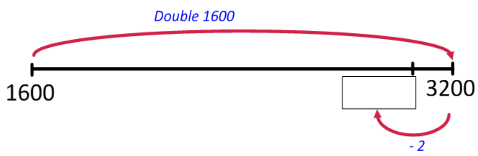
N.B. There is no explicit reference to mental calculation strategies in the programmes of study for Year 4 in the national curriculum. However, with an overall aim for fluency, appropriate mental strategies should always be considered before resorting to formal written procedures, with the emphasis on pupils making their own choices from an increasingly sophisticated range of strategies.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving fractions and decimals to two decimal places

Y4 Addition & Subtraction

Strategies & Guidance	Representations
Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000. *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.* **Count forwards and backwards in tenths and hundredths**	  Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. E.g. 990 + 10 or 19.9 + 0.1
Using known facts and knowledge of place value to derive facts. **Add and subtract multiples of 10, 100 and 1000 mentally** *Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.*	 2 + 4 = 6 20 + 40 = 60 200 + 400 = 600 2000 + 4000 = 6000
Adding and subtracting by partitioning one number and applying known facts. *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*	See Year 3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.

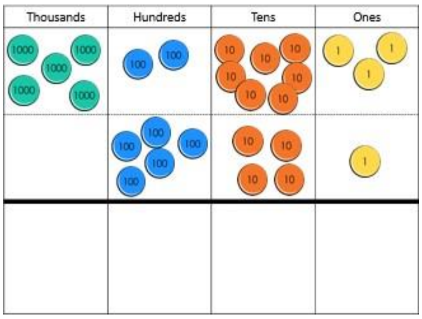
Strategies & Guidance	Representations
Round and adjust *Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.* *It is very easy to be confused about how to adjust and so visual Representations and logical reasoning are essential to success with this strategy.* *Build flexibility by completing the same calculation in a different order.*	3527 + 296 = 3827 - 4  Completing the same calculation but adjusting first: 3527 + 296 = 3523 + 300  4523 – 3997 = 523 + 3  Completing the same calculation but adjusting first: 4523 – 3997 = 4526 - 4000 
Near doubles *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.*	1600 + 1598 = double 1600 – 2 

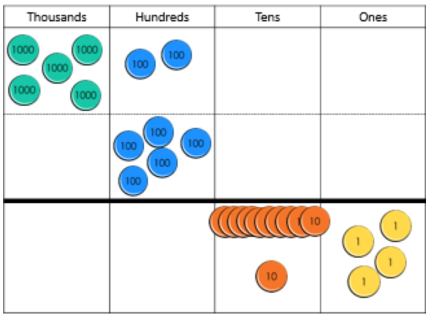
Strategies & Guidance	Representations

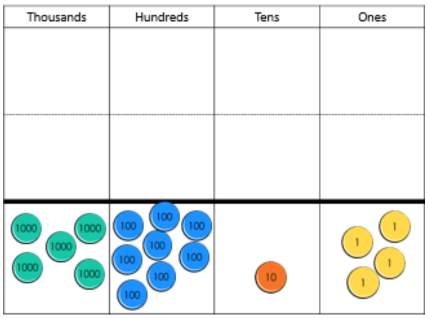
Written column methods for addition

Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.

This method and the language to use are best understood through the PD videos available on MyMastery.







	5	2	7	3
+		5	4	1
		1		
	5	8	1	4

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Written column methods for subtraction

Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.

This method and the language to use are best understood through the PD videos available on the MyMastery.

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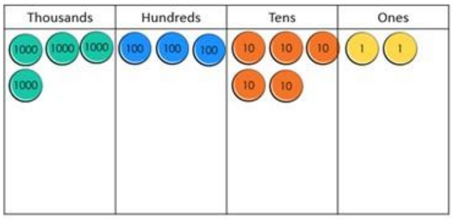

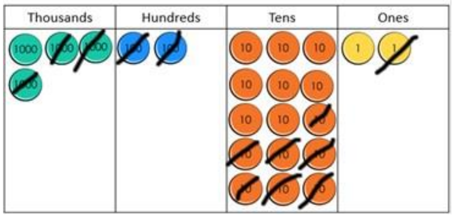

$$
\begin{array}{r}
4\;{}^{2}\!\not{3}\;{}^{1}5\;2\\
-\;3\;2\;7\;1\\
\hline
1\;0\;8\;1\\
\hline
\end{array}
$$

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Calculating with decimal numbers

Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.

Place value counters are another useful manipulative for representing decimal numbers.

All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.

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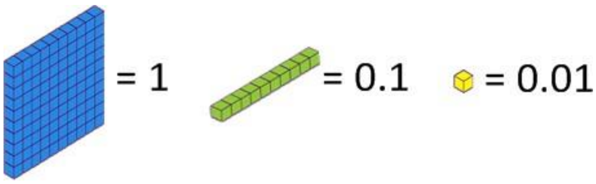
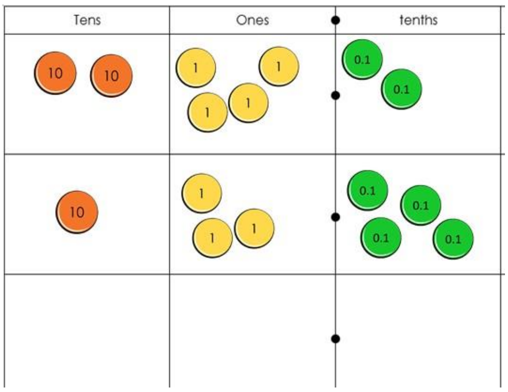


24.2 + 13.4 =



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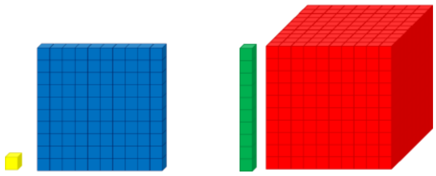
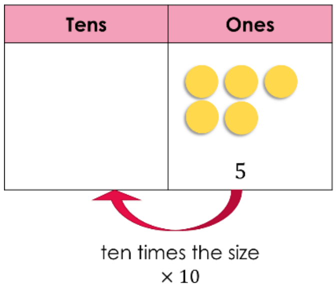
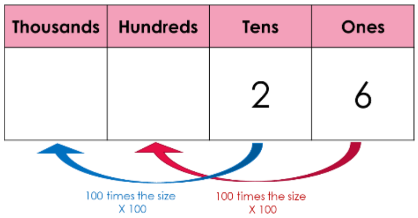
National Curriculum objectives linked to multiplication and division

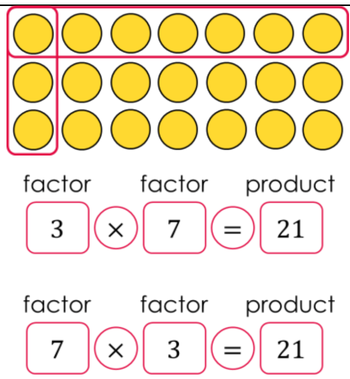
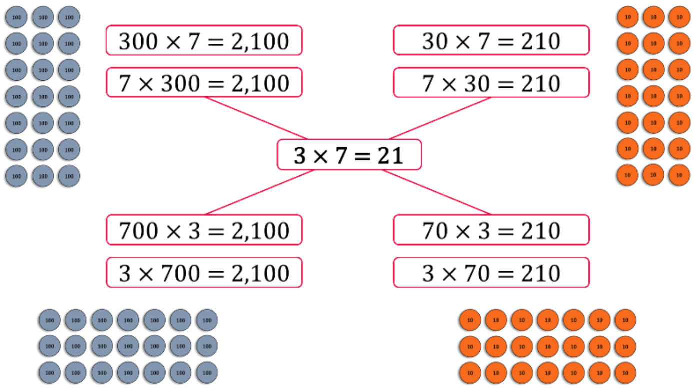
These objectives are explicitly covered through the strategies outlined in this document:

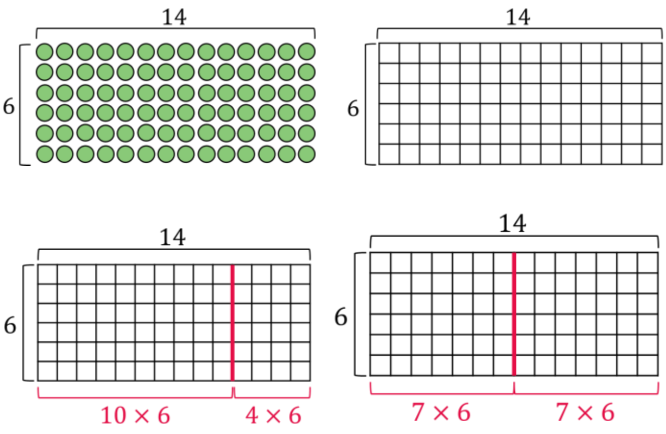
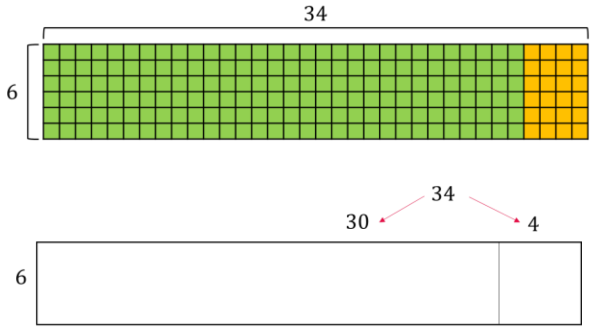
- count from 0 in multiples of 6, 7, 9, 25 and 1000
- recall and use multiplication and division facts for multiplication tables up to 12 × 12
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- recognise and use factor pairs and commutativity in mental calculations
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as *n* objects are connected to *m* objects.

Strategies & Guidance	Representations
Multiplying by 10 and 100 *Pupils begin to think about multiplication as scaling. When you multiply whole numbers by 10 and 100 this is equivalent to making a number 10 or 100 times the size.* *When you multiply by ten, each part is ten times the size. The ones become tens, the tens become hundreds, etc.* *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.* *Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000.*	 *One hundred is one hundred times the size of one one.* *One thousand is one hundred times the size of one ten.*  *Five made ten times the size is 50.* *50 is ten times the size of five.* *Five multiplied by ten is 50.*  *26 made 100 times the size is 2,600.* *26 multiplied by 100 is equal to 2,600.* *First, we had 26 ones. Now we have 26 hundreds.*

Strategies & Guidance	Representations
Using known facts and place value for mental multiplication involving multiples of 10 and 100 *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.*	 ***Factors** are numbers that are multiplied together to make another number.* *A **product** is the number made when other numbers are multiplied.*  *If I know that three ones multiplied by seven ones is equal to 21, then I know that three ones multiplied by seven tens is equal to 210.* *One of the factors is ten times greater, so the product is ten times greater.*

Strategies & Guidance	Representations
Multiplying by partitioning one number and multiplying each part *Pupils build on mental multiplication strategies and develop an explicit understanding of the distributive law of multiplication.* *They begin to multiply a two-digit number by a one-digit number by splitting arrays and area models. They recognise that factors can be partitioned in ways other than into '10 and a bit'.* *They begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.* This illustrates the distributive property of multiplication: $a \times (b + c) = a \times b + a \times c$ and $a \times (b - c) = a \times b - a \times c$	14×6  34×6  $34 \times 6 = 30 \times 6 + 4 \times 6$ $= 180 + 24$ $= 204$

<table>
<tr><th>Strategies & Guidance</th><th>Representations</th></tr>
<tr><td>

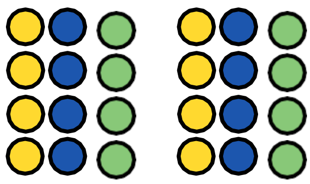
Mental multiplication of three 1-digit numbers, using the associative law

Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.

</td><td>

Four pots each containing two flowers which each have seven petals. How many petals in total?

$(4 \times 2) \times 7$ or $4 \times (2 \times 7)$



$3 \times 4 \times 2$

Three groups of four, two times

Multiplication can be done in any order.
The order of the factors does not alter the product.

</td></tr>
<tr><td>

Short multiplication of a 2-digit number by a 1-digit number

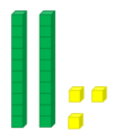
To begin with, pupils are presented with calculations that require no regrouping and then progress to regrouping from the ones to the tens. They learn how to use the expanded written algorithm alongside Dienes blocks to support their conceptual understanding. They then build on, and apply their understanding to the compact written algorithm.

</td><td>

Expanded layout **Compact layout**

		2	3
	×		3
			9
	+	6	0
		6	9

		2	3
	×		3
		6	9



× 3

If there are ten or more ones, we regroup the ones into tens and ones.

If there are ten or more tens, we regroup the tens into hundreds and tens.

</td></tr>
</table>

Strategies & Guidance	Representations

Short multiplication of 3-digit number by 1-digit number

To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.

With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.

		5	1	2
×				3
				6
			3	0
	1	5	0	0
	1	5	3	6

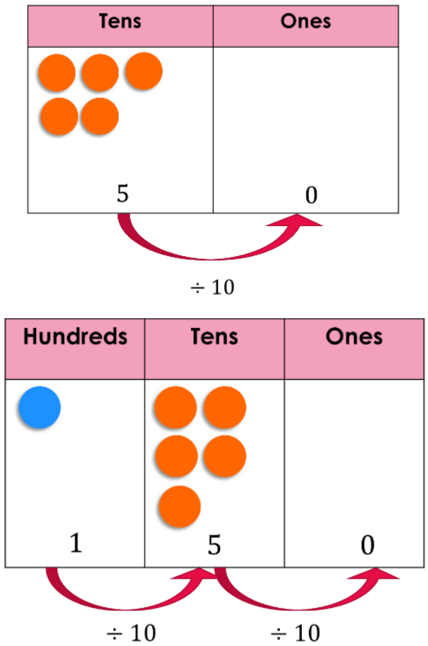
To calculate 512 × 3, represent the number 512. Multiply each part by 3, regrouping as needed.

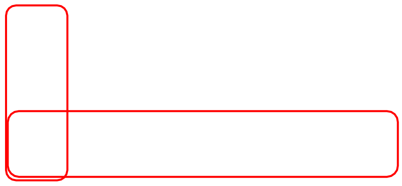
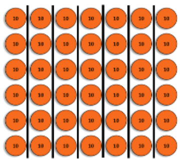
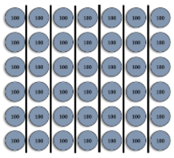
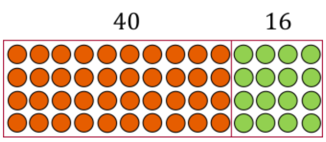
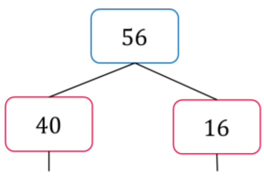
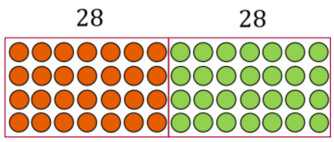
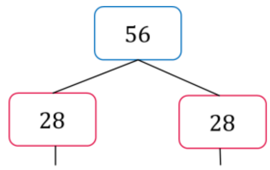
		5	1	2
×				3
	1	5	3	6

When we multplly by zero, the product is zero.

<table>
<tr><th>Strategies & Guidance</th><th>Representations</th></tr>
<tr><td>Dividing by 10 and 100

When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.

When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21</td><td>
I‘m making 150 one-hundredth the size.
This is the same as dividing by 100.</td></tr>
</table>

Strategies & Guidance	Representations
Derived facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	*3 × 7 and 21 ÷ 3*  *If I know 42 ÷ 7 = 6, then I know:* 7 groups of 6  420 ÷ 7 = 600 7 × 600 = 420 6 groups of 70  420 ÷ 70 = 6 70 × 6 = 420 7 groups of 6  4,200 ÷ 7 = 600 7 × 600 = 4,200 6 groups of 700  4,200 ÷ 700 = 6 700 × 6 = 4,200
Division of 2-digit numbers by a 1-digit number *Pupils use their place-value knowledge to divide a two-digit number by a one-digit number through partitioning the two-digit number into tens and ones, dividing the parts by the one-digit number, then adding the partial quotients. Pupils then progress to partitioning the two-digit number into multiples of the divisor.*	56 ÷ 4   40 ÷ 4 = 10 16 ÷ 4 = 4 10 + 4 = 14 56 ÷ 4 = 14   28 ÷ 4 = 7 28 ÷ 4 = 7 7 + 7 = 14 56 ÷ 4 = 14

<table>
<tr><th>Strategies & Guidance</th><th>Representations</th></tr>
<tr><td>

Short division of 2-digit numbers by a 1-digit number

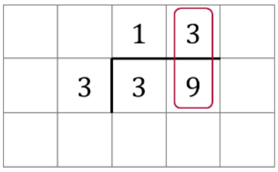
Pupils start with dividing 2-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used to model the algorithm and help pupils relate it to what they already know about division and to develop conceptual understanding.

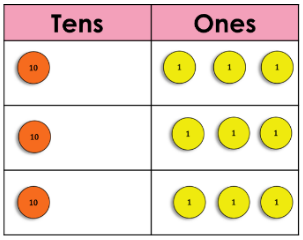
They progress to calculations that require regrouping in the tens column.

Pupils learn that division is the only operation for which the formal algorithm begins with the most significant digit (on the left).

</td><td>

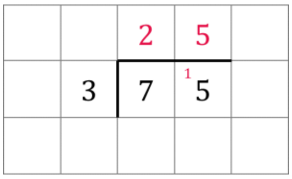
39 ÷ 3

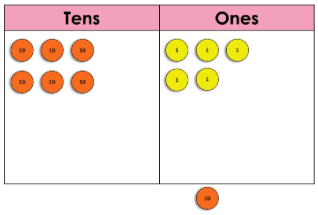


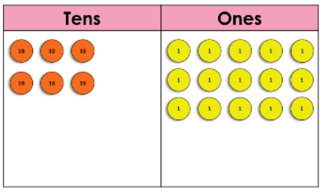


75 ÷ 3

Two groups of three tens can be made from seven tens. There is one ten remaining.







One ten can be regrouped for ten ones, making 15 ones altogether.

Five groups of three ones can be made from 15 ones, with no ones remaining.
75 divided by three is equal to 25.

</td></tr>
</table>

Strategies & Guidance	Representations

Short division of a 3-digit number by a 1-digit number

Pupils use place value counters alongside the written method of short division, beginning with examples that do not involve regrouping and progressing to multiple regrouping.

Pupils recognise that no regrouping is required when the dividend has digits that are multiples of the divisor.

Pupils progress to short division where the dividend has digits smaller than the divisor.

726 ÷ 6

	1	2	1	
6	7	$^{1}2$	6	

7 hundreds ÷ 6 = 1 hundred remainder 1 hundred
1 hundred = 10 tens
plus 2 more tens = 12 tens
12 tens ÷ 6 = 2 tens
6 ones ÷ 6 = 1 one

438 ÷ 6

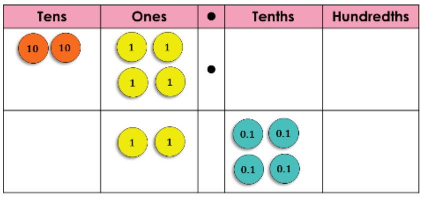
	0	7	3	
6	4	$^{4}3$	$^{1}8$	

4 hundreds ÷ 6 = 0 remainder 4 hundreds
4 hundreds = 40 tens
plus 3 more tens = 43 tens
43 tens ÷ 6 = 7 tens remainder 1 ten
1 ten = 10 ones
plus 8 more ones = 18 ones
18 ones ÷ 6 = 3 ones

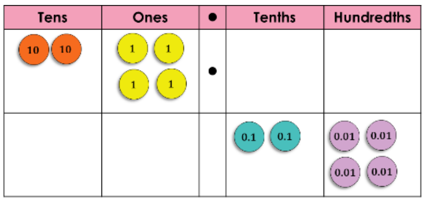
Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.

24 ÷ 10 = 2.4



24 ÷ 100 = 0.24



Progression in calculations
Year 5 + Year 6

Year 5 and Year 6 are together because the calculation strategies used are broadly similar, with Year 6 using larger and smaller numbers. Any differences for Year 6 are highlighted in red.

National Curriculum objectives linked to integer addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- add and subtract numbers mentally with increasingly large numbers
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- use negative numbers in context, and calculate intervals across zero
- perform mental calculations, including with mixed operations and large numbers
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

Y5 and Y6 Addition & Subtraction

<table>
<tr><th>Strategies & Guidance</th><th>Representations</th></tr>
<tr>
<td>
Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000

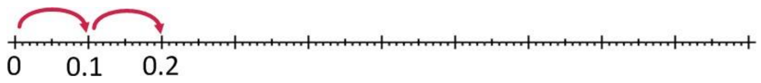
Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.

In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.

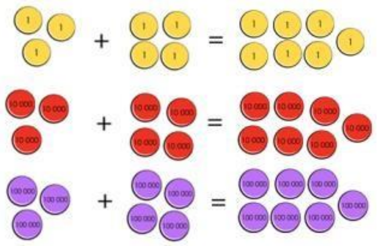
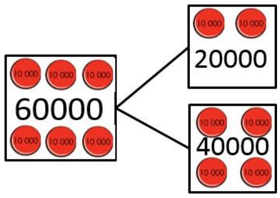
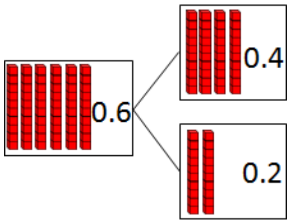
In Year 6 pupils work with numbers up to 10 000 000.
</td>
<td>
Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.

<table>
<tr><th>Hundred Thousands</th><th>Ten Thousands</th><th>Thousands</th><th>Hundreds</th><th>Tens</th><th>Ones</th><th>tenths</th><th>hundredths</th><th>thousandths</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Counting sticks and number lines:

Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes.
e.g. 9900 + 100 = 10 000 or 99 000 + 1000 = 100 000
</td>
</tr>
</table>

Strategies & Guidance	Representations
Using known facts and understanding of place value to derive *Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.* *In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.* *In Year 6 extend to multiples of one million.* *These derived facts should be used to estimate and check answers to calculations.*	 3 + 4 = 7 30 000 + 40 000 = 70 000 300 000 + 400 000 = 700 000 20 000 + 40 000 = 60 000 40 000 + 20 000 = 60 000 60 000 - 40 000 = 20 000 60 000 - 20 000 = 40 000   0.6 = 0.2 + 0.4 0.6 = 0.4 + 0.2 0.2 = 0.6 - 0.4 0.4 = 0.6 - 0.2

Strategies & Guidance	Representations

Partitioning one number and applying known facts to add.

Pupils can use this strategy mentally or with jottings as needed.

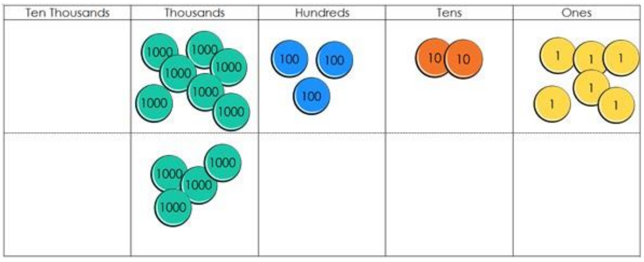
Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.

They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.

Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

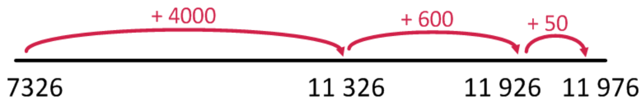
Partitioning into place value amounts (canonical partitioning):

4650 + 7326 = 7326 + 4000 + 600 + 50



With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added.

Represent pictorially with an empty numberline:



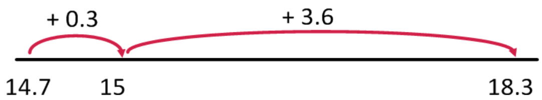
Partitioning in different ways (non-canonical partitioning):

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10.

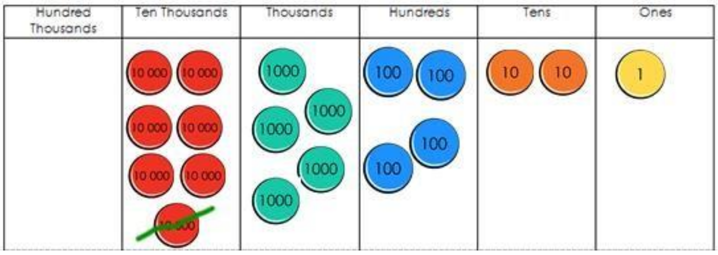
6785 + 2325 = 6785 + 15 + 200 + 2110

The strategy can be used with decimal numbers, Make one:

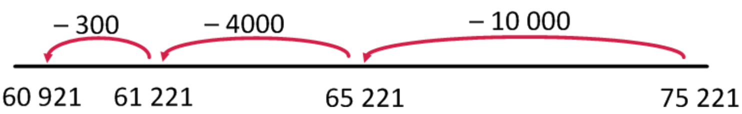
14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3



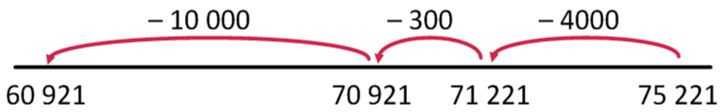
Strategies & Guidance	Representations
Subtraction by partitioning and applying known facts. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 75 221 – 14 300 = 75 221 – 10 000 – 4000 – 300



Represent pictorially with a number line, starting on the right and having the arrows jump to the left:

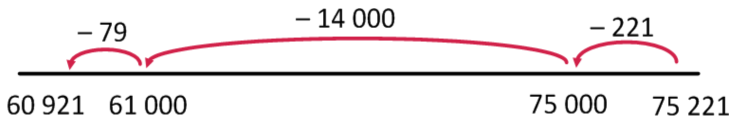


Develop understanding that the parts can be subtracted in any order and the result will be the same:



Partitioning in different ways (non-canonical partitioning):

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.



<table>
<tr><th>Strategies & Guidance</th><th>Representations</th></tr>
<tr><td>

Calculate difference by “counting back”

It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.

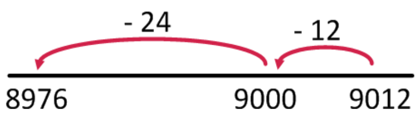
</td><td>

75 221 – 14 300

Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.

Finding the difference is efficient when the numbers are close to each other:

9012 – 8976



</td></tr>
<tr><td>

Calculate difference by “counting on”

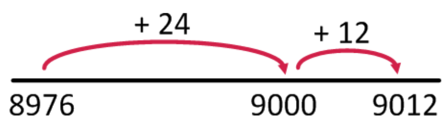
Addition strategies can be used to find difference.

</td><td>

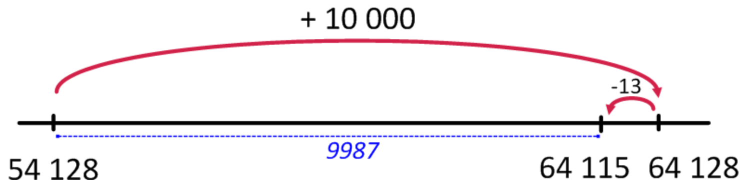
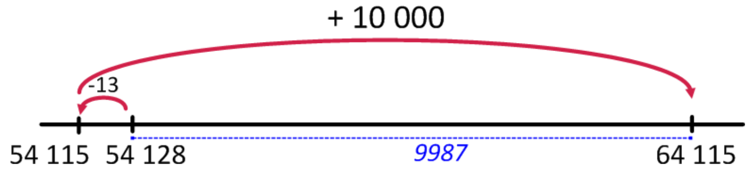
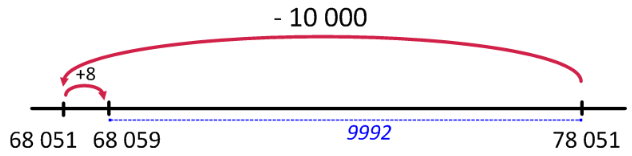
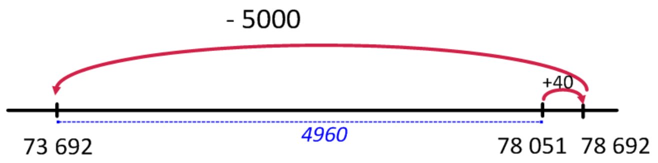
75 221 – 14 300

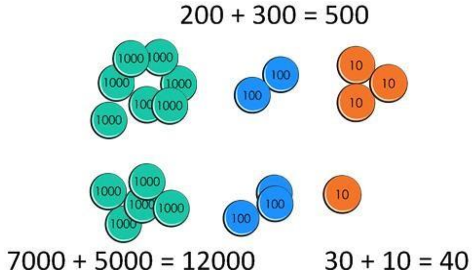
Finding the difference is efficient when the numbers are close to each other

9012 – 8976



</td></tr>
</table>

Strategies & Guidance	Representations
Round and adjust **Addition and subtraction using compensation** *Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.* *It is very easy to be confused about how to adjust and so visual Representations and logical reasoning are essential to success with this strategy.*	**Addition**  54 128 + 9987 = 54 128 + 10 000 – 13 = 64128 - 13 Pupils should realise that they can adjust first:  54128 + 9987 = 54128 – 13 + 10 000 = 54 115 + 10 000 **Subtraction**  78 051 – 9992 = 78 051 – 10 000 + 8 = 68 051 + 8 Pupils should realise that they can adjust first:  78 051 – 4960 = 78 051 + 40 – 5000 = 78 692 - 5000
Near doubles *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.*	160 + 170 = double 150 + 10 + 20 160 + 170 = double 160 + 10 or 160 + 170 = double 170 - 10 2.5 + 2.6 = double 2.5 + 0.1

Strategies & Guidance	Representations
Partition both numbers and combine the parts *Pupils should be secure with this method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.* *If multiple regroupings are required, then pupils should consider using the column method.*	7230 + 5310 = 12 000 + 500 + 40  Pupils should be aware that the parts can be added in any order.

Strategies & Guidance	Representations

Written column methods for addition

In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.

Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.

Continue to use concrete manipulatives alongside the formal method.

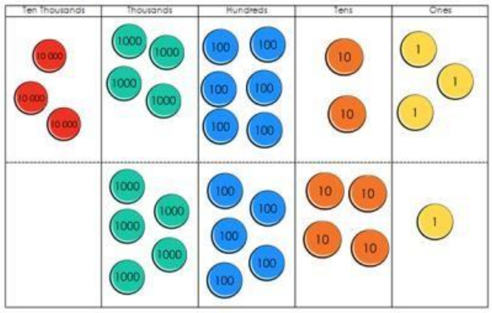
When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.

Exemplification of this method and the language to use are best understood through viewing the PD videos available on MyMastery.

For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value.

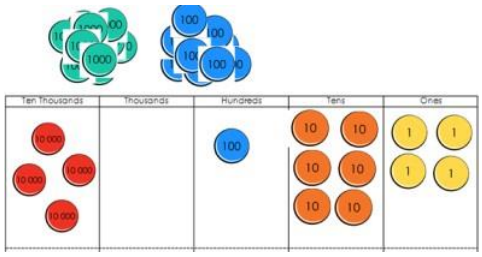
```
    3 4 6 2 3
+     5 5 4 1
-------------

-------------
```

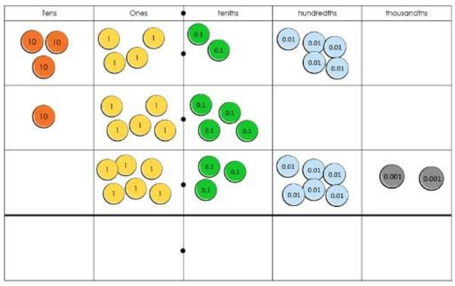


Combine the counters in each column and regroup as needed:

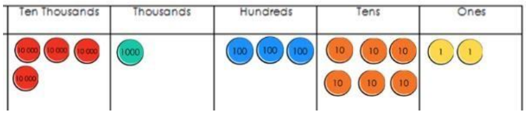
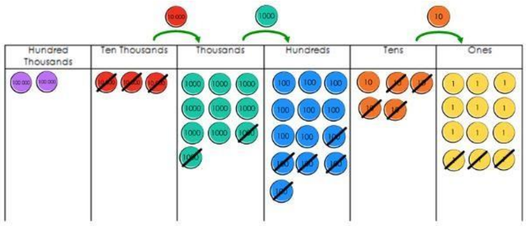
```
    3 4 6 2 3
+     5 5 4 1
    1 1
-------------
    4 0 1 6 4
-------------
```



Decimal numbers:




```
    3 4 . 2 5
    1 5 . 4
+     6 . 3 6 2
    1 1   1
---------------
    5 6 . 0 1 2
---------------
```

Strategies & Guidance	Representations
Written column methods for subtraction *In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.* *Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.* *In Year 3 and 4 they become more familiar with calculations that require ‘regrouping to regroup’. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.* *Exemplification of this method and the language to use are best understood through viewing the PD videos available on MyMastery.*	4 1 3 6 2 - 3 2 2 4 3  3 1 5 1 4 1 3 6 2 - 3 2 2 4 3 9 1 1 9  The term regrouping should be the language used. You can use the terms ‘exchange’ with subtraction but it needs careful consideration. You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones). Or you can ‘exchange’ one of the tens for 10 ones resulting in 5 tens and 12 ones. If you have exchanged, then the number has been regrouped.

Progression in calculations

Year 5 + Year 6

National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- multiply and divide whole numbers by 10, 100 and 1000
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

Strategies & Guidance	Representations
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000	Ruby walked 130 m. Her mum walked 100 times as far. How far did Ruby's mum walk?

Strategies & Guidance

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

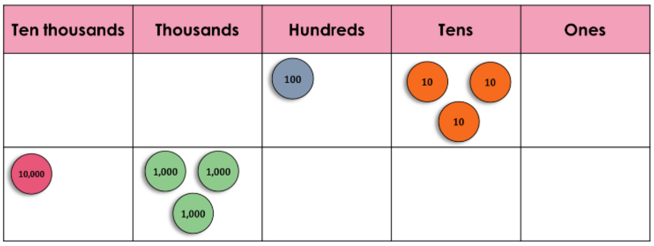
Through the context of measures, pupils learn to multiply and divide whole numbers by 10, 100 and 1,000 alongside place value counters and charts.

Avoid saying that you "add a zero" when multiplying by 10, 100 and 1,000 and instead use the language of place holder.

Use place value counters and charts to visualise and then notice what happens to the digits.

Representations

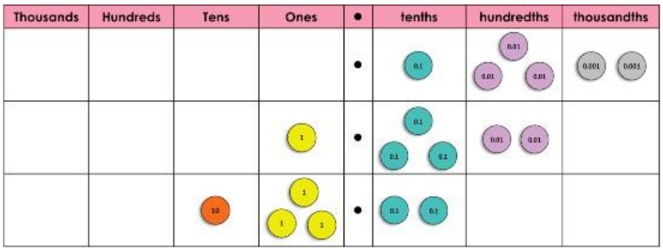
Ruby walked 130 m. Her mum walked 100 times as far. How far did Ruby's mum walk?



13,000 m is one hundred times as far as 130 m.

When you multiply by one hundred, each part is ten times the size. The ones become hundreds, the tens become thousands, etc.

To find the inverse of one hundred times as many, divide by one hundred.



0.132

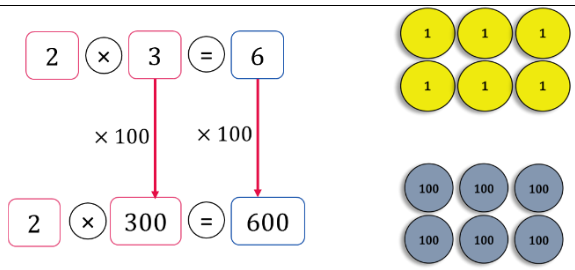
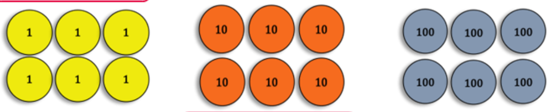
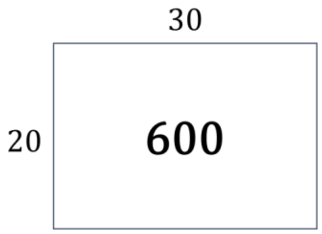
1.32

13.2

1.32 ÷ 10 = 0.132
0.132 is one-tenth the size of 1.32.

13.2 ÷ 100 = 0.132
0.132 is one-hundredth the size of 13.2

When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.

Strategies & Guidance	Representations
Using known facts and place value to derive multiplication facts *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity is further extended and applied to find a range of related facts.* *Pupils should work with decimals with up to two decimal places.* *These derived facts should be used to estimate and check answers to calculations.*	 *If one factor is made one hundred times the size, the product will become one hundred times the size.* $30 \times 2 = 60$ $300 \times 2 = 600$ $2 \times 30 = 60$ $2 \times 300 = 600$ $2 \times 3 = 6$ $20 \times 3 = 60$ $200 \times 3 = 600$ $3 \times 2 = 6$ $3 \times 20 = 60$ $3 \times 200 = 600$  $6 \div 2 = 3$ $60 \div 2 = 30$ $600 \div 2 = 300$ $6 \div 3 = 2$ $60 \div 20 = 3$ $600 \div 200 = 3$ $60 \div 3 = 20$ $600 \div 300 = 2$ $60 \div 30 = 2$ $600 \div 3 = 200$  *If both factors are made ten times the size, the product will be 100 times the size.*

Strategies & Guidance	Representations

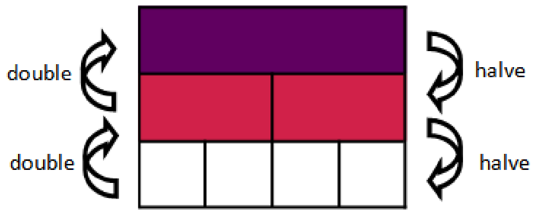
These are the multiplication facts pupils should be able to derive from a known fact.

2 100 000		700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000		70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000		7000 x 3	700 x 30	70 x 300	7 x 3000		
2100		700 x 3	70 x 30	7 x 300			
210		70 x 3	7 x 30				
21	=	**7 x 3**					
2.1		0.7 x 3	7 x 0.3				
0.21		0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021		0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

Doubling and halving

Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.

Doubling and halving can then be used in larger calculations.

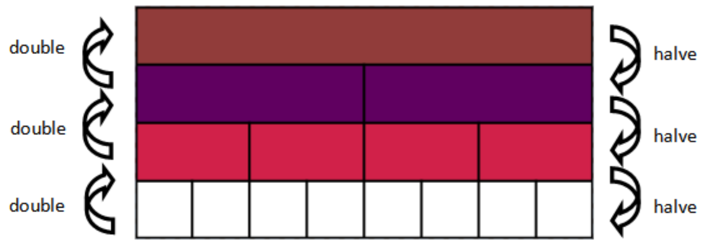


Multiply by 4 by doubling and doubling again

e.g. 16 × 4 = 32 × 2 = 64

Divide by 4 by halving and halving again

e.g. 104 ÷ 4 = 52 ÷ 2 = 26

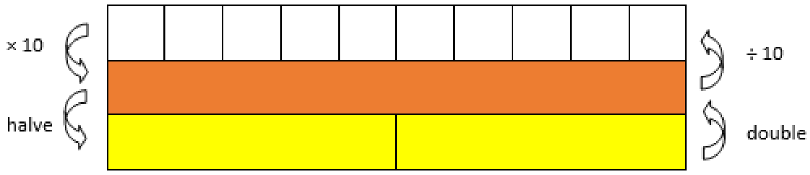
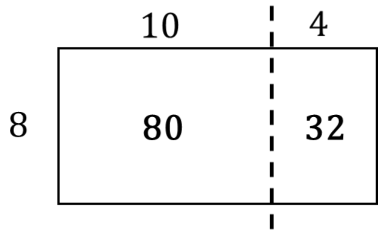
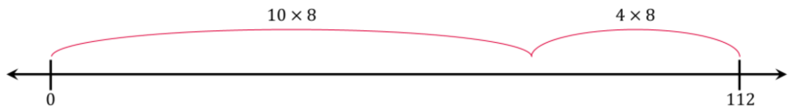


Multiply by 8 by doubling three times

e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96

Divide by 8 by halving three times

e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13

Strategies & Guidance	Representations
	 **Multiply by 5** by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90. **Divide by 5** by dividing by 10 and doubling, e.g. 460 ÷ 5 = double 46 = 92
Multiply by partitioning one number and multiplying each part **Distributive law** **a × (b + c) = a × b + a × c** Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:	8 × 14 = 8 × 10 + 8 × 4  Represent with area model  Jottings on a number line

Strategies & Guidance	Representations

Using knowledge of factors

Pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.

Pupils will be using the commutative and associative laws of multiplication.

Commutative law

a × b = b × a

Associative law

a × b × c = (a × b) × c

= a × (b × c)

They should explore and compare the different options and choose the most efficient order to complete calculations.

Calculate 6 × 24 by using factor pairs of 24

Two and twelve are factors of 24:

6 × 2 × 12

6 × 12 × 2

Three and eight are factors of 24:

6 × 3 × 8

6 × 8 × 3

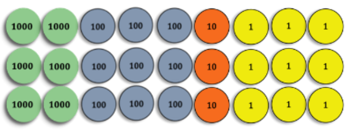
Four and six are factors of 24:

6 × 4 × 6

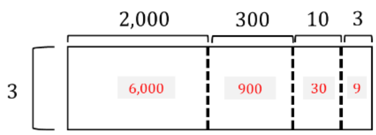
6 × 6 × 4

Multiplying 3- or 4-digit number by a 1-digit number using the formal written method of short multiplication

Conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.



×	2,000	300	10	3
3	6,000	900	30	9



	2	3	1	3
×				3
	6	9	3	9

Strategies & Guidance	Representations

Multiplying by a 2-digit number

Formal written method of long multiplication

In Year 5 pupils are extended from multiplication by a 1-digit number to multiplication by a 2-digit number.

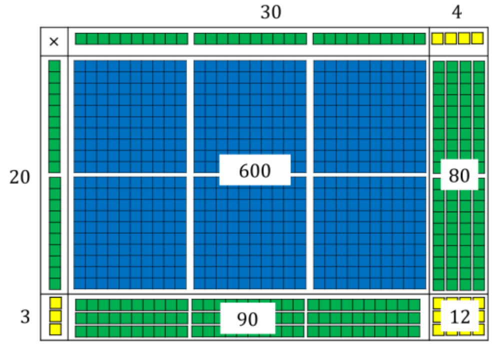
Extend understanding of the distrubitive law to develop conceptual understanding of the two rows of the formal written method.

Dienes blocks can be used to construct area models to represent this.

The grid method is used alongside the formal written method to strengthen understanding of partitioning and place value in long multiplication.

Representations:

34 × 23



×	30	4
20	600	80
4	90	12

			1	2
			9	0
			8	0
+		6	0	0

42 × 23

×	40	2
20	800	40
3	120	6

	H	T	O		
		4	2		
×		2	3		
	1	2	6		(42 × 3)
+	8	4	0		(42 × 20)
	9	6	6		

Multiplying a 3- or 4-digit number by a 2-digit number.

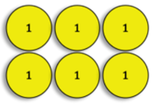
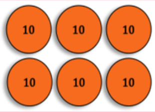
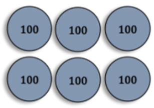
Grid method and formal written method of long multiplication.

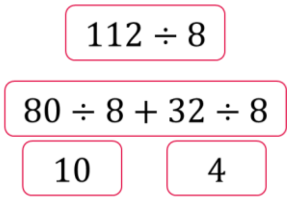
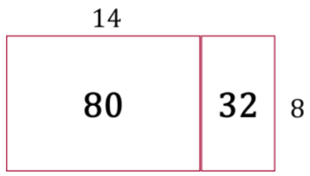
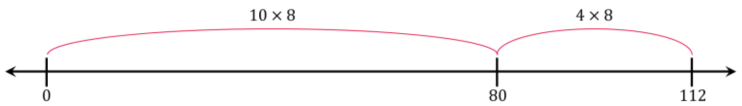
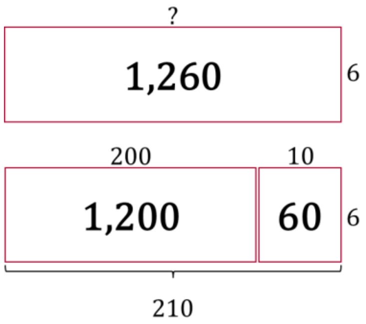
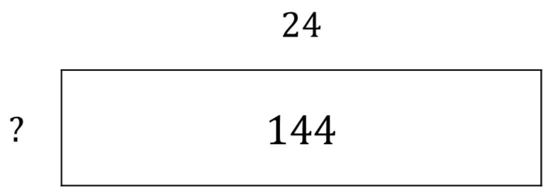
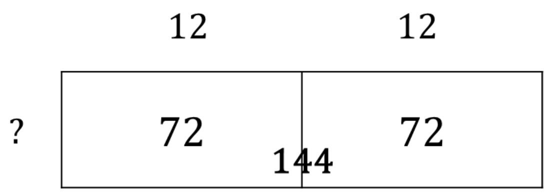
Representations:

124 × 26

×	100	20	4
20	100 × 20 = 2,000	20 × 20 = 400	4 × 20 = 80
6	100 × 6 = 600	20 × 6 = 120	4 × 6 = 24

		1	2	4	
×			2	6	
		7	4	4	(124 × 6)
+	2	4	1 8	2 0	(124 × 20)
	3	2	2	4	(124 × 26)
	1	1			

Strategies & Guidance	Representations
Deriving facts from known facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	$15 \div 3 = 5$ $\times 100$ ↓ $\times 100$ ↓ $1{,}500 \div 3 = 500$ *If the dividend is made one hundred times the size, the quotient will be one hundred times the size.* $30 \times 2 = 60$ $300 \times 2 = 600$ $2 \times 30 = 60$ $2 \times 300 = 600$ $2 \times 3 = 6$ $20 \times 3 = 60$ $200 \times 3 = 600$ $3 \times 2 = 6$ $3 \times 20 = 60$ $3 \times 200 = 600$    $6 \div 2 = 3$ $60 \div 2 = 30$ $600 \div 2 = 300$ $6 \div 3 = 2$ $60 \div 20 = 3$ $600 \div 200 = 3$ $60 \div 3 = 20$ $600 \div 300 = 2$ $60 \div 30 = 2$ $600 \div 3 = 200$

Strategies & Guidance	Representations
Using knowledge of multiples to divide *Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).*	112 ÷ 8 = 80 ÷ 8 + 32 ÷ 8    1260 ÷ 6 = 1200 ÷ 6 + 60 ÷ 6 
Using knowledge of factors to divide *Pupils explore this strategy when using repeated halving.* *2 x 2 = 4 and so if you divide by 4 the same result can be achieved by dividing by two and then by two again.*	144 ÷ 24  *I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.* *144 ÷ 2 ÷ 12* 

Strategies & Guidance	Representations
Short division	8528 ÷ 4

Dividing a 4-digit numbers by 1-digit numbers

The thought process of the traditional algorithm is as follows:

How many 4s in eight? Two
How many 4s in five? One with 1 remaining so regroup.
How many 4s in 12? three

Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.

How many 4s in 8000? 2000
How many 4s in 500?
100 with one remaining (illogical) The answer would be 125.

Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.

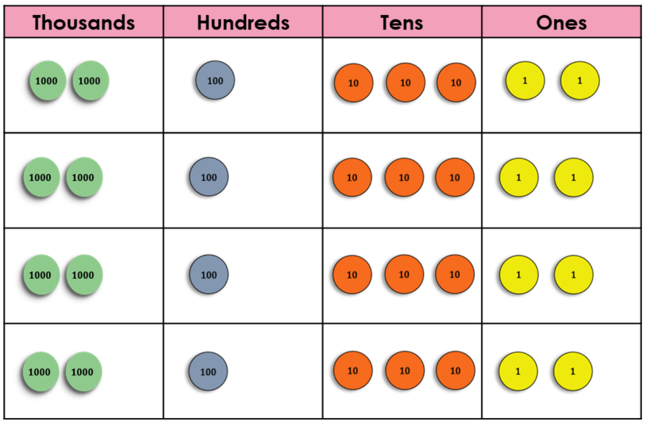
Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the short division algorithm.

Area models are also useful representations, as seen with other strategies and exemplified for long division.

8528 ÷ 4

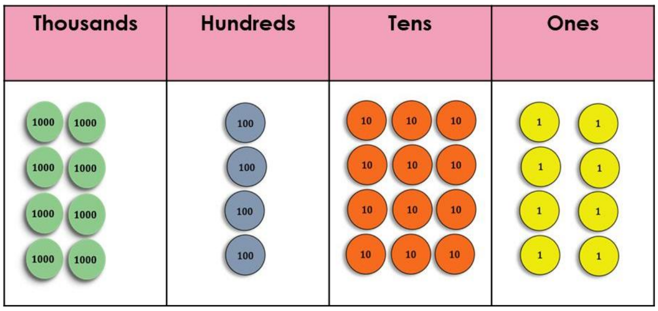
	2	1	3	2
4	8	5	^{1}2	8

Sharing



Eight thousands shared into four equal groups
Five hundreds shared into ten tens
12 tens shared into four equal groups
Eight ones shared into four equal groups.

Grouping



How many groups of four thousands in eight thousands?
How many groups of four hundreds in five hundreds?
Regroup one hundred for ten tens.
How many groups of four tens in 12 tens?
How many groups of four ones in eight ones?

Strategies & Guidance	Representations
Long division **Dividing a 4-digit number by a 2-digit number** *Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.* *If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.*