Additive reasoning

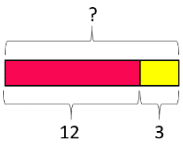
Change structures

augmentation (increasing)

where an existing value has been added to

Standard

First there were 12 people on the bus. Then three more people got on. How many people are on the bus now?

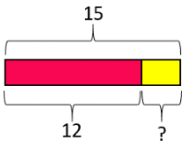


"I know both parts. My first part is twelve and my second part is three. I don't know the whole. I need to add the parts of twelve and three to find the whole."

12 + 3 = ?

Non-standard

First there were 12 people on the bus and then some more people got on at the school. Now there are 15 people on the bus. How many people got on at the school?

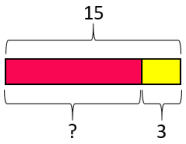


"I know my first part is twelve and I know the whole is 15. I don't know the value of the second part. To find the second part, I could add on from 12 to make 15 or I could subtract 12 from 15."

12 + ? = 15 15 – 12 = ?

Non-standard

First there were some people on the bus then it stopped to pick up three more passengers at the bank. Altogether now there are 15 people on the bus. How many were people were on the bus before it stopped atthe bank?



"I know the value of the second part is three and that the whole is 15. I don't know the value of the first part. To find the first part, I could add on from three to 15 or I could subtract three from 15."

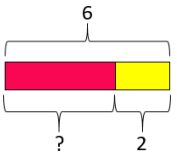
? + 3 = 15 15 – 3 = ?

reduction (decreasing)

where an existing value has been reduced

Standard

First Kieran had six plates in his cupboard. Then he took two plates out to use for dinner. How many plates are in the cupboard now?

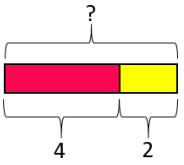


"I know the whole is six. I know one of the part that has been taken away is two. I don't know the other part. I need to subtract the known part, two, from the whole, six, to find the remaining part."

6 – 2 = ? 2 + ? = 6

Non-standard

First there were some plates in the cupboard. Then Kieran took two out for dinner. Now there are four left. How many plates were in the cupboard to start with?

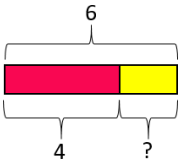


"I know the part that has been taken away is two and the part that is left is four. I don't know the whole. I can find the whole by adding the parts of four and two."

? – 2 = 4 2 + 4 = ?

Non-standard

First there were six plates in the cupboard. Then Kieran took some out for dinner. There are now four plates left in the cupboard. How many did Kieran take out?



"I know the whole is six and the remaining part is four. I don't know the part that was taken away. To find the part that was taken away I can add on from four to make six or I could subtract four from six."

6 – ? = 4 6 – 4 = ?

Note: the 'first… then… now' structure is used heavily in KS1 to scaffold pupils' understanding of change structures. Once pupils are confident with the structures, such linguistic scaffolding

can be removed, and question construction can be changed to expose pupils to a greater range of nuance in interpreting problems. For example, the second and third reduction problems could be reworded as follows:

Kieran took two plates out of his cupboard for dinner. There were four left. How many plates were in the cupboard to begin with?

There were six plates in the cupboard before Kieran took some out for dinner. If there were four plates left in the cupboard, how many did Kieran take out?

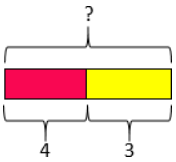
These present the same knowns and unknowns, and therefore the same bar models and resulting equations to solve the problems; however, the change in wording makes them more challenging to pupils who have only worked with a 'first… then… now' structure so far.

Part-whole structures

Combination (aggregation)/partitioning

combining two or more discrete quantities/splitting one quantity into two or more sub-quantities

Hakan and Sally have made a stack of their favourite books. Four books belong to Hakan, three to Sally. How many books are in the stack altogether?

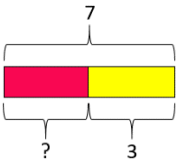


"I know both parts. One part is four and the other part is three. I don't know the whole.I need to add the parts of three and four to find the whole."

4 + 3 = ? 3 + 4 = ?

(Only one problem has been written for combination as, owing to the commutativity of addition, the only change in question wording would be to swap Hakan and Sally's names. The resulting bar model and calculation would be identical.)

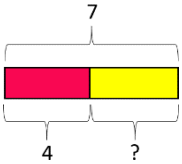
Sally and Hakan have made a stack of their favourite books. There are seven books altogether. If three of them are Sally's, how many belong to Hakan?



"I know the whole is seven and that one of the parts is three. I don't know the other part. I need to add on from three to make seven or subtract three from seven to find the other part."

3 + ? = 7 7 – 3 = ?

Sally and Hakan have made a stack of their favourite books. There are seven books altogether. If four of them are Hakan's, how many belong to Sally?



"I know the whole is seven and that one of the parts is four. I don't know the other part. I need to add on from four to make seven or subtract four from seven to find the other part."

4 + ? = 7 7 – 4 = ?

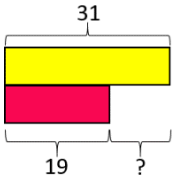
Note: all part-whole contexts are considered to be 'standard', as the language of part-whole is unambiguous.

Comparison structures

Comparison structures involve a relationship between two quantities; their relationship is expressed as a difference. The structures vary by which of the values are known/unknown (the larger quantity, the smaller quantity and/or their difference). Part-whole language is not used here because the context contains not one single 'whole', but instead two separate quantities and it is the relationship between them being considered. Comparison bar models are therefore used to model these structures, which are known to be the most challenging for pupils to interpret.

Smaller quantity and larger quantity are known (comparative difference)

Standard



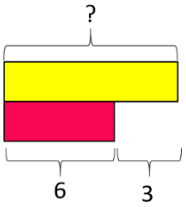
*Navin has saved £19 from his pocket money. Sara has saved £31 from her pocket money. How much **more** has Sara saved than Navin? **or** How much **less** has Navin saved than Sara?*

"I know one quantity is 19 and the other quantity is 31. I don't know the difference. To find the difference I could add on from 19 to make 31 or I could subtract 19 from 31."

$19 + ? = 31$ $31 - 19 = ?$

Smaller quantity and difference are known (comparative addition)

Standard

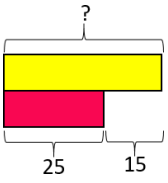


*Ella has six marbles. Robin has three **more** than Ella. How many marbles does Robin have?*

"I know the smaller quantity is six. I know the difference is three. I don't know the larger quantity. To find the larger quantity I need to add three to six."

$6 + 3 = ?$

Non-standard



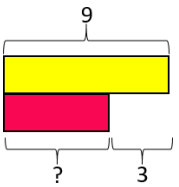
*Samir and Lena are baking shortbread but Lena's recipe uses 15g **less** butter than Samir's. If Lena needs to use 25g of butter, how much does Samir need?*

"I know the smaller quantity is 25. I know the difference between the quantities is 15. I don't know the larger quantity. To find the larger quantity I need to add 15 to 25."

$? - 15 = 25$ $25 + 15 = ?$

Larger quantity and difference are known (comparative subtraction)

Non-standard

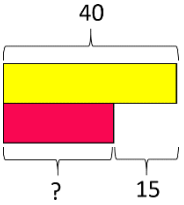


*Ella has some marbles. Robin has three **more** than Ella and he has nine marbles in total. How many marbles does Ella have?*

"I know the larger quantity is nine. I know the difference between the quantities is three. I don't know the smaller quantity. To find the smaller quantity I need to add on from three to make nine or subtract three from nine."

$? + 3 = 9$ $9 - 3 = ?$

Standard



*Samir's shortbread recipe uses 40g of butter. Lena's recipe uses 15g **less** butter. How much butter does Lena need?*

"I know one quantity is 40. I know the difference between the quantities is 15. I don't know the smaller quantity but I know it is 15 less than 40. To find the smaller quantity, I need to subtract 15 from 40."

$40 - 15 = ?$ $? + 15 = 40$

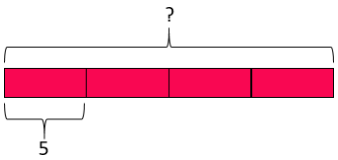
Multiplicative reasoning

Repeated grouping structures

repeated addition

groups (sets) of equal value are combined or repeatedly added

There are four packs of pencils. Each contains five pencils. How many pencils are there?



"I know there are four equal parts and that each part has a value of five. I don't know the value of the whole. To find the whole, I need to multiply four and five."

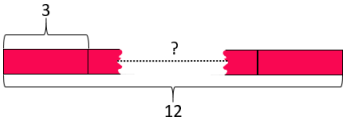
5 + 5 + 5 + 5 = ?

5 × 4 = ?

repeated subtraction (grouping)

groups (sets) of equal value are partitioned from the whole or repeatedly subtracted

There are 12 counters. If each child needs three counters to play the game, how many children can play?



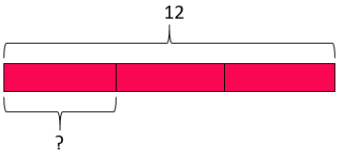
"I know the whole is twelve and that the value of each equal part is three. To find the number of equal parts, I need to know how many threes are in twelve."

3 × ? = 12 12 ÷ 3 = ?

sharing (into equal groups)

the whole is shared into a known number (must be a positive integer) of equal groups (sets)

Share twelve counters equally between three children. How many counters does each child get?



"I know the whole is twelve and the number of equal parts is three. I don't know the value of each part. To find the value of each part, I need to know what goes into twelve three times."

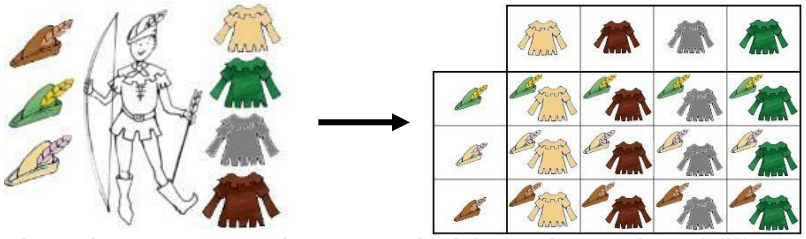
? × 3 = 12 12 ÷ 3 = ?

Cartesian product of two measures

correspondence

calculating the number of unique combinations that can be created from two (or more) sets

Robin has three different hats and four different tops. How many different outfits can he create, that combine one hat and one top?



"I know how many hats there are, and I know how many tops there are. I don't know the number of different outfits that can be created. To find the number of outfits, I need to find how many different tops can be worn with each hat or how many different hats can be worn with each top."

4 × 3 = ? 3 × 4 = ?

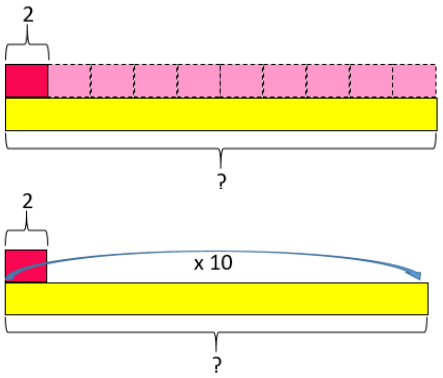
Scaling structures

Continuous structure

scaling up
(‘times the size/times as much’)

the original value is increased by a given scale factor

Rita receives £2 pocket money every week. Sim earns ten times as much money for her paper round. How much money does Sim earn?



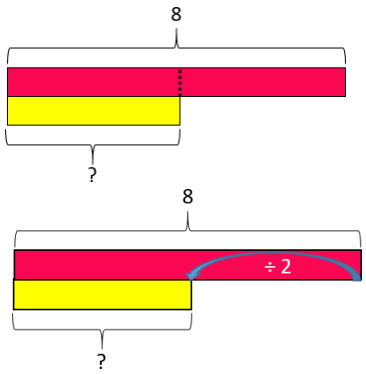
“I know one value is two and I know the second value is ten times the size. I don’t know the second value. To find the second value, I need to multiply two by ten.”

2 × 10 = ?

scaling down
(‘times smaller/one tenth of the size’)

the original value is reduced by a given scale factor

The house in my model village needs to be half the height of the church. If the church is 8 cmtall, how tall does the house need to be?



“I know one value is eight and I know the second value is half as great. I don’t know the second value. To find the second value, I need to halve eight (or divide it by two).”

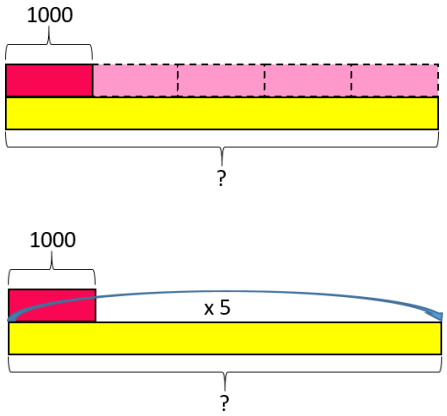
Half of 8 is ? 8 ÷ 2 = ?

Discrete structures

scaling up (‘times as many’)

the value of the original quantity is increased by a given scale factor

The Albert Hall can hold five times as many people as the Festival Hall. If the Festival Hall holds 1000 people, how many does the Albert Hall hold?



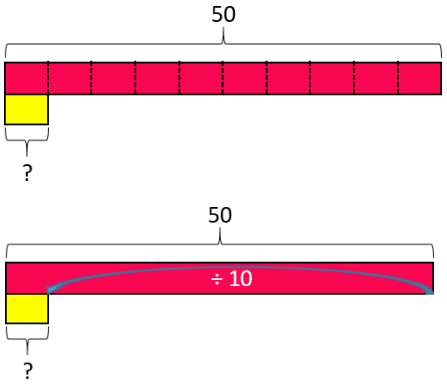
“I know one value is 1000 and I know the second value is five times the size. I don’t know thesecond value. To find the second value, I need tomultiply 1000 by five.”

1000 × 5 = ?

scaling down (‘times fewer’)

the value of the original quantity is decreased by a given scale factor

Anouska’s garden pond has ten times fewer frogs than fish. If there are fifty fish, how many frogs are there?



“I know one value is 50 and I know the second value is one tenth of the size. I don’t know the second value. To find the second value, I need to divide fifty by ten.”

50 ÷ 10 = ?